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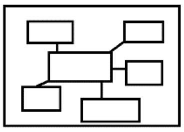
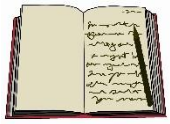
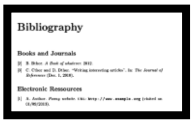
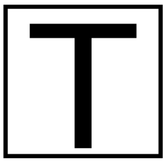
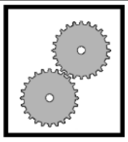
MATHEMATICS

2025 LAST PUSH

GRADE 12

GUIDE FOR TEACHERS AND LEARNERS

ICON DESCRIPTION

 MIND MAP	 EXAMINATION GUIDELINE	 CONTENTS	 ACTIVITIES
 BIBLIOGRAPHY	 TERMINOLOGY	 WORKED EXAMPLES	 STEPS

CONTENTS PAGE

SECTION 1: Paper 1

Content and Examination Guidelines

ALGEBRA, EQUATIONS AND INEQUALITIES

Working with fractions

- Multiplication and division of fractions

  Example 1

  Simplify

  $\frac{5}{2} \times \frac{2}{3} = \frac{5\times2}{2\times3} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$

  Example 2

  $\frac{5}{2} \div \frac{2}{3} = \frac{5}{2} \times \frac{3}{2} = \frac{5\times3}{2\times2} = \frac{15}{4} = 3\frac{3}{4}$

- Addition and Subtraction of fractions

  Example 1

  Simplify

  $\frac{5}{2} + \frac{2}{3} = \frac{5}{2} \times \frac{3}{3} + \frac{2}{3} \times \frac{2}{2} = \frac{15}{6} + \frac{4}{6} = \frac{15+4}{6} = \frac{19}{6} = 3\frac{1}{6}$

  Example 2

  $\frac{5}{2} - \frac{2}{3} = \frac{5}{2} \times \frac{3}{3} - \frac{2}{3} \times \frac{2}{2} = \frac{15}{6} - \frac{4}{6} = \frac{15-4}{6} = \frac{11}{6} = 1\frac{5}{6}$

Exponents and Surds

Laws of exponents

$(a, b > 0 \; and \; m, n \in Z)$

Law	Example
$a^m a^n = a^{m+n}$	$2^3 2^4 = 2^{3+4} = 2^7 = 128$
$(a^m)^n = a^{mn}$	$(2^3)^4 = 2^{3.4} = 2^{12} = 4096$
$(ab)^n = a^n b^n$	$(20)^3 = (2.10)^3 = 2^3.10^3 = 8.1000 = 8000$
$(\frac{a}{b})^n = \frac{a^n}{b^n}$	$(\frac{2}{5})^3 = \frac{2^3}{5^3} = \frac{8}{125}$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^5}{2^3} = 2^{5-3} = 2^2 = 4$

N.B $\quad a^0 = 1 \quad [a \neq 0]$

$\quad a^{-n} = \frac{1}{a^n} \quad [a \neq 0]$

Surds

Simplify expressions and solve equations using the laws of exponents for rational exponents where

$$x^{\frac{p}{q}} = \sqrt[q]{x^p}\,;\ x > 0; q > 0$$

Take note that:

- $\sqrt{x} = x^{\frac{1}{2}}$
- $\sqrt[3]{x} = x^{\frac{1}{3}}$
- $\sqrt[n]{x} = x^{\frac{1}{n}}$

Exponential equations

The unknown is in the exponent, e.g. In the equation:

$$2^x = 8, \textit{the unknown } x \textit{ is in the exponent}$$

N.B $a^x > 0 \quad [a > 0 \textit{ and } a \neq 1]$

Example 1

$2^x = 8$

Solution

$2^x = 2^3$

$x = 3$

Example 2

$2^{2x+1} + 15.2^x = 8$

Solution

$2^{2x+1} + 15.2^x = 8$

$\therefore 2^{2x}.\ 2^1 + 15.2^x = 8$

$\therefore 2(2^x)^2 + 15(2^x) - 8 = 0$

$\therefore (2.2^x - 1)(2^x + 8) = 0$

$\therefore 2.2^x = 1 \text{ or } 2^x = -8$

$\therefore 2^x = \frac{1}{2}$ *or* no solution

$\therefore 2^x = 2^{-1}$

$\therefore x = -1$

Alternatively:

$2^{2x}.2^1 + 15.2^x = 8$

Let $k = 2^x$

$\therefore k^2 = 2^{2x}$

$\therefore 2k^2 + 15k - 8 = 0$

$\therefore (2k-1)(k+8) = 0$

$\therefore k = \frac{1}{2}$ or $k = -8$

$\therefore 2^x = \frac{1}{2}$ or $2^x = -8$

$\therefore x = -1$ no solution

Equations with rational exponents

Example 1

Solve for x

$$3x^{\frac{2}{5}} - 5x^{\frac{1}{5}} - 2 = 0$$

Solution

$3\left(x^{\frac{1}{5}}\right)^2 - 5x^{\frac{1}{5}} - 2 = 0$

$let\ k = x^{\frac{1}{5}}$

$3k^2 - 5k - 2 = 0$

$(3k+1)(k-2) = 0$

$k = -\frac{1}{3}$ or $k = 2$

$x^{\frac{1}{5}} = -\frac{1}{3}$

$\left(x^{\frac{1}{5}}\right)^5 = \left(-\frac{1}{3}\right)^5$

$x = -\frac{1}{243}$

Quadratic Equations

Solving equations of the form $\boldsymbol{ax^2 + bx + c = 0}$

- Solving quadratic equations by factorisation

  Example

  Solve for x:

  Solution

  $x^2 + 5x + 6 = 0$

  $(x + 2)(x + 3) = 0$

  $x = -2 \ \ or \ \ x = -3$

- Solving quadratic equations by using the general quadratic formula

  General quadratic formula is given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

  Example

  Solve for x:

  $x^2 + 2x - 5 = 0$

  Solution

  $a = 1, b = 2, c = -5$

  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

  $x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-5)}}{2(1)}$

  $x = \frac{-2 \pm 2\sqrt{6}}{2}$

  $x = \frac{-2 + 2\sqrt{6}}{2}$ or $x = \frac{-2 - 2\sqrt{6}}{2}$

  $x = -1 + \sqrt{6}$ or $x = -1 - \sqrt{6}$

  $x = 1{,}45$ or $x = -3{,}45$ [Rounded to TWO decimal places]

Equations With Fractions

Example 1

Solve for x

$$\frac{x^2+x-2}{x-1}+1=0$$

Solution

Restrictions: $\boldsymbol{x-1\neq 0}$ **,** $\boldsymbol{\therefore x\neq 1}$

$\frac{x^2+x-2}{x-1}+1=0\ \therefore$

$x^2+x-2+x-1=0$

$\therefore x^2+2x-3=0$

$\therefore (x+3)(x-1)=0$

$\therefore x=-3$ or $x=1$ But $x\neq 1$

$\therefore x=-3$

Alternatively:

$$\frac{x^2+x-2}{x-1}+1=0$$

$$\therefore \frac{(x+2)(x-1)}{(x-1)}+1=0$$

$$\therefore x+2+1=0$$

$$\therefore x=-3$$

Example 2

Solve for x

$$\frac{2}{4x^2-1}+\frac{1}{1-2x}=\frac{x}{2x+1}$$

Solution

Restrictions: $x \neq \frac{1}{2}$ ***and*** $x \neq -\frac{1}{2}$

$$\frac{2}{4x^2-1}+\frac{1}{1-2x}=\frac{x}{2x+1}$$
$$\therefore \frac{2}{(2x+1)(2x-1)}-\frac{1}{(2x-1)}=\frac{x}{(2x+1)}$$
$$\therefore 2-(2x+1)=x(2x-1)$$
$$\therefore 2-2x-1=2x^2-x$$
$$\therefore 0=2x^2+x-1$$
$$\therefore 0=(2x-1)(x+1)$$
$$\therefore x \neq \frac{1}{2} \quad \text{or} \quad x=-1$$

Using k-method to solve equations

You may use the k-method to solve equations with repeated expressions

Example 1

Solve for x

$$\sqrt{x^2+x+10}=x^2+x-2$$

Solution

$$\sqrt{x^2+x+10}=x^2+x-2$$
$$\text{Let } k=x^2+x$$
$$\therefore \sqrt{k+10}=k-2$$
$$\therefore k+10=k^2-4k+4$$
$$\therefore 0=k^2-5k-6$$
$$\therefore 0=(k-6)(k+1)$$
$$\therefore k=6 \quad \text{or} \quad k \neq -1$$

$$\text{But } k \neq -1$$
$$\therefore k=6$$
$$\therefore x^2+x=6$$
$$\therefore x^2+x-6=0$$
$$\therefore (x+3)(x-2)=0$$
$$\therefore x=-3 \quad \text{or} \quad x=2$$

Example 2

Solve for a

$\frac{a^2-a-3}{a^2-a}+a^2-a+1=\frac{5}{a^2-a}$

Solution

$Let\ a^2-a=k$

$\frac{k-3}{k}+k+1=\frac{5}{k}$

$k-3+k^2+k=5$

$k^2+2k-8=0$

$(k-2)(k+4)=0$

$k=2\quad or\quad k=-4$

$let\ k=a^2-a$

$a^2-a=2\quad or\quad a^2-a=-4$

$a^2-a-2=0\quad or\quad a^2-a+4=0$

$(a-2)(a+1)=0\quad or\quad a=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(4)}}{2(1)}$

$a=2\ or\ a=-1\qquad or\quad a=\frac{1\pm\sqrt{-15}}{2}$

No Real Solution

Equations With Surds

When we can't simplify a number to remove a square root (or cube root etc.) then it is a surd.

When there are square roots in an equation, isolate the surd part, then square both sides to get rid of the square root. You may have introduced an extra solution, which is not valid, when you squared both sides, therefore it is important to check all your solutions if they satisfy the original equation.

Example 1

Solve for x

$x-\sqrt{7-3x}+1=0$

Solution

$x-\sqrt{7-3x}+1=0$

$\therefore x+1=\sqrt{7-3x}$

$\therefore x^2+2x+1=7-3x$

$\therefore x^2+5x-6=0$

$\therefore (x+6)(x-1)=0$

$\therefore x=-6 \quad \text{or} \quad x=1$

But $x\neq-6$

$\therefore x=1$

Example 2

Solve for x

$\sqrt{x-2}-\frac{6}{\sqrt{x-2}}=1$

Solution

$\sqrt{x-2}-\dfrac{6}{\sqrt{x-2}}=1$

Let $k=\sqrt{x-2}$

$\therefore k-\dfrac{6}{k}=1$

$\therefore k^2-6=k$

$\therefore k^2-k-6=0$

$\therefore (k-3)(k+2)=0$

$\therefore k=3 \quad \text{or} \quad k=-2$

$\therefore \sqrt{x-2}=3 \quad \text{or} \quad \sqrt{x-2}=-2$

$\therefore x-2=9$ no real solution

$\therefore x=11$

Alternatively:

$\sqrt{x-2}-\dfrac{6}{\sqrt{x-2}}=1$

$\therefore (\sqrt{x-2})^2-6=\sqrt{x-2}$

$\therefore x-2-6=\sqrt{x-2}$

$\therefore x-8=\sqrt{x-2}$

$\therefore (x-8)^2=x-2$

$\therefore x^2-16x+64=x-2$

$\therefore x^2-17x+66=0$

$\therefore (x-11)(x-6)=0$

$\therefore x=11 \quad \text{or} \quad x=6$

But $x\neq 6$

$\therefore x=11$

Quadratic Inequalities

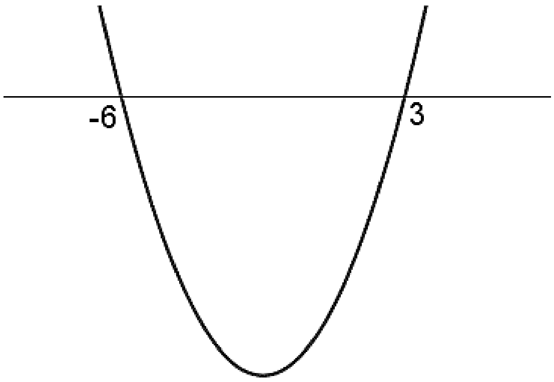
The form:

$$ax^2 + bx + c < 0 \quad or \quad ax^2 + bx + c > 0 \quad or \quad ax^2 + bx + c \leq 0 \quad or \quad ax^2 + bx + c \geq 0$$

Example 1

$(x+1)(x+2) < 20$

Solution



$(x+1)(x+2) < 20$

$\therefore x^2 + 3x + 2 < 20$

$\therefore x^2 + 3x - 18 < 0$

$\therefore (x+6)(x-3) < 0$

$\therefore -6 < x < 3$

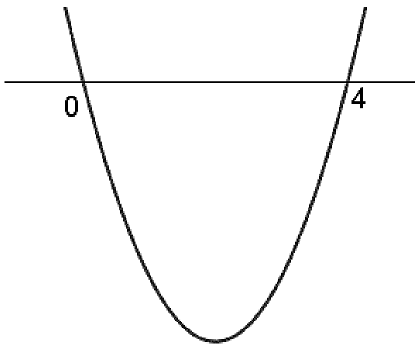
Or interval notation - $\boldsymbol{x \in (-6; 3)}$

Example 2

Solve for x

$\frac{x^2}{4} \geq x$

Solution



$\frac{x^2}{4} \geq x$

$\therefore x^2 \geq 4x$

$\therefore x^2 - 4x \geq 0$

$\therefore x(x-4) \geq 0$

$\therefore x \leq 0 \quad \text{or} \quad x \geq 4$

Or interval notation - $\boldsymbol{x \in (-\infty; 0] \cup [4; \infty)}$

Simultaneous Equations

Equations in two unknowns, one of which is linear and the other quadratic.

Example 1

Solve for x and y:

$$x-3y+4=0 \quad \text{and} \quad 3+xy-x^2=y^2$$

Solution

$x-3y+4=0$

$\therefore x=3y-4$

$3+(3y-4)y-(3y-4)^2=y^2$

$\therefore 3+3y^2-4y-(9y^2-24y+16)=y^2$

$\therefore 3+3y^2-4y-9y^2+24y-16=y^2$

$\therefore 0=7y^2-20y+13$

$\therefore 0=(7y-13)(y-1)$

$\therefore y=\frac{13}{7} \quad \text{or} \quad y=1$

$\therefore x=3\left(\frac{13}{7}\right)-4 \quad \text{or} \quad x=3(1)-4$

$x=\frac{11}{7} \quad \text{or} \quad x=-1$

Example 2

Solve for x and y:

$$x+y=5$$
$$xy=21-x^2-y^2$$

Solution

$x + y = 5$

$\therefore y = 5 - x$

$\therefore x(5 - x) = 21 - x^2 - (5 - x)^2$

$\therefore 5x - x^2 = 21 - x^2 - (25 - 10x + x^2)$

$\therefore 5x - x^2 = 21 - x^2 - 25 + 10x - x^2$

$\therefore x^2 - 5x + 4 = 0$

$\therefore (x - 1)(x - 4) = 0$

$\therefore x = 1 \quad \text{or} \quad x = 4$

$\therefore y = 4 \quad \text{or} \quad y = 1$

Nature of Roots: Quadratic equations

The nature of roots depends on the value of the discriminant, Δ.

$\boldsymbol{\Delta = b^2 - 4ac}$	**Roots**
$\Delta < 0$	Non-real
$\Delta \geq 0$	Real
$\Delta > 0$ *and complete square*	Real, Rational and Unequal
$\Delta > 0$ *and not a complete square*	Real, Irrational and Unequal
$\Delta = 0$	Real, Rational and Equal

Nature of roots **Graphs**

$\Delta < 0$

Roots are non-real.

There are no x-intercepts.

$\Delta = 0$

Roots are real and equal.

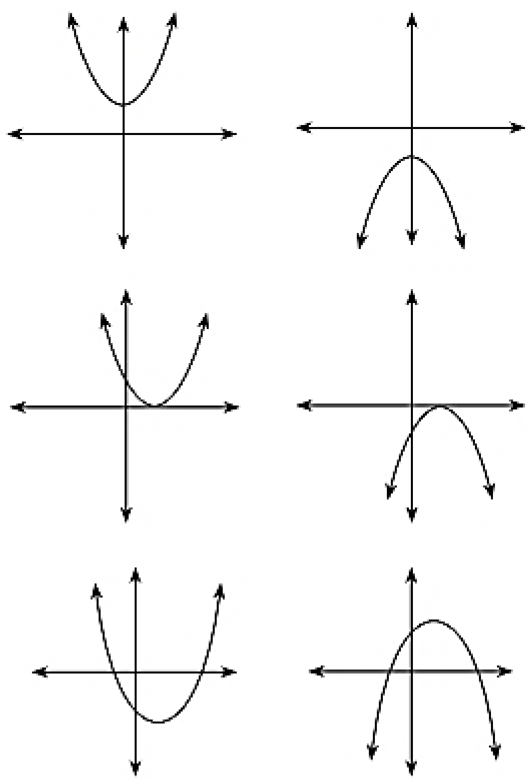
There is only one x-intercept and it is at the turning point of the graph.

$\Delta > 0$

Roots are real and unequal (two roots):

If Δ is a squared rational number, roots are rational.

If Δ is not a squared number, the roots are irrational.



Example 1

Without solving the equation $x^2 - 2x - 6 = 0$, determine the nature of its roots.

Solution

$x^2 - 2x - 6 = 0$

$\Delta = (-2)^2 - 4(1)(-6)$

$\therefore \Delta = 28$

Real, irrational and unequal.

Example 2

The roots of a quadratic equation are $x = \dfrac{-2 \pm \sqrt{13 - 2k}}{3}$.

a. For which values of k will the roots be real?

b. Determine the smallest positive integral value of k for which the solutions will be rational.

Solution

a.

$$13 - 2k \geq 0$$
$$\therefore -2k \geq -13$$
$$\therefore k \leq \frac{13}{2}$$
$$\therefore k \leq 6\frac{1}{2}$$

b. $k = 2$

Examination Guidelines (Algebra, Equations and Inequalities)

Source: Mathematics Examination Guidelines Grade 12, 2021

1. Solving quadratic equations by completing the square will NOT be examined.
2. Solving quadratic equations using the substitution method (k-method) is examinable.
3. Equations involving surds that lead to a quadratic equation are examinable.
4. Solution of non-quadratic inequalities should be seen in the context of functions.
5. Nature of the roots will be tested intuitively with the solution of quadratic equations and in all the prescribed functions.

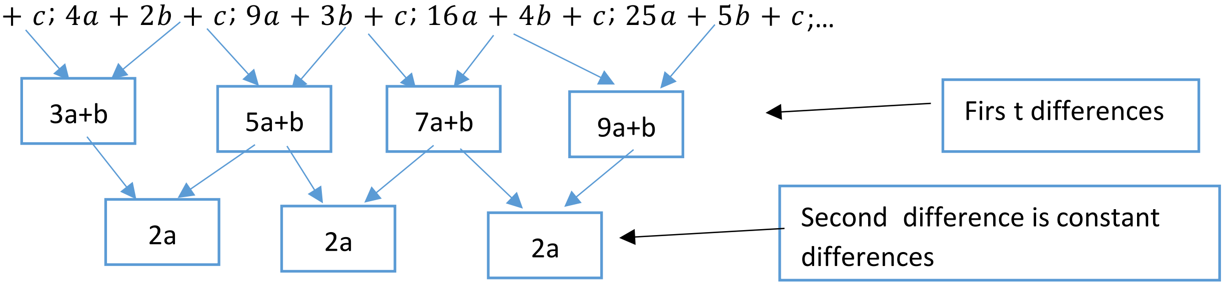
PATTERNS AND SEQUENCES

N.B $n \in N$

Quadratic Number Patterns

The general formular: $T_n = an^2 + bn + c$ For any quadratic number pattern:

$a + b + c;\ 4a + 2b + c;\ 9a + 3b + c;\ 16a + 4b + c;\ 25a + 5b + c; \ldots$



N.B First differences of a quadratic number pattern form an arithmetic sequence

Arithmetic Sequence And Series

General term of an **arithmetic sequence**: $T_n = a + (n - 1)d$

Where: a - first term of the sequence

d - common/constant difference

n – position of a term / number of terms

T_n – n^{th} term / last term

Sum formular for an **arithmetic series**: $S_n = \frac{n}{2}[2a + (n - 1)d]$

OR

$S_n = \frac{n}{2}[a + l]$ where l is the last term

Arithmetic Series Proof:

$S_n = a + [a + d] +. . . +[a + (n - 2)d] + [a + (n - 1) d] . . . (1)$
rewrite equation (1) in reverse:
$S_n = [a + (n - 1)d] + [a + (n - 2)d]+. . . +[a + d] + a . . . (2)$

Adding equation (1) and equation (2)
$2S_n = [2a + (n - 1)d] + [2a + (n - 1)d]+. . . +[2a + (n - 1)d] + [2a + (n - 1)d]$
$2S_n = n \times [2a + (n - 1)d]$
$\therefore S_n = \frac{n}{2}[2a + (n - 1)d]$

Geometric Sequence and Series

General term of a **geometric sequence**: $T_n = ar^{n-1}$

Where: a - first term of the sequence

r - common/constant ratio

n – position of a term / number of terms

T_n – n^{th} term

Sum formular for a **geometric series**: $S_n = \frac{a(r^n-1)}{r-1}$ $;r \neq 1$

Proof:

$S_n = a + ar + ar^2 +\ldots+ ar^{n-2} + ar^{n-1} \ldots . . (1)$

multiply equation (1) by r:
$rS_n = \quad ar + ar^2 + ar^3 +\ldots+ ar^{n-1} + ar^n \ldots\ldots(2)$

Subtract equation (1) from equation (2)
$rS_n - S_n = -a + ar^n$
$rS_n - S_n = ar^n - a$
$S_n(r - 1) = a(r^n - 1)$
$S_n = \frac{a(r^n - 1)}{r - 1} \quad ;r \neq 1$

SUM TO INFINITY

Formular: $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

Sum to infinity exists for a convergent geometric series.

Convergence: $-1 < r < 1$

SIGMA NOTATION

$$\sum_{k=a}^{b} T_k$$

- T_k is the general term
- $number\ of\ terms = Top - Bottom + 1$

$$= b - a + 1$$

FOR ANY SUM FORMULAR

$$T_n = S_n - S_{n-1}$$

Examination Guidelines (Patterns and Sequences)

Source: Mathematics Examination Guidelines Grade 12, 2021

1. The sequence of first differences of a quadratic number pattern is linear. Therefore, knowledge of linear patterns can be tested in the context of quadratic number patterns.
2. Recursive patterns will not be examined explicitly.
3. Links must be clearly established between patterns done in earlier grades.

FUNCTIONS AND GRAPHS

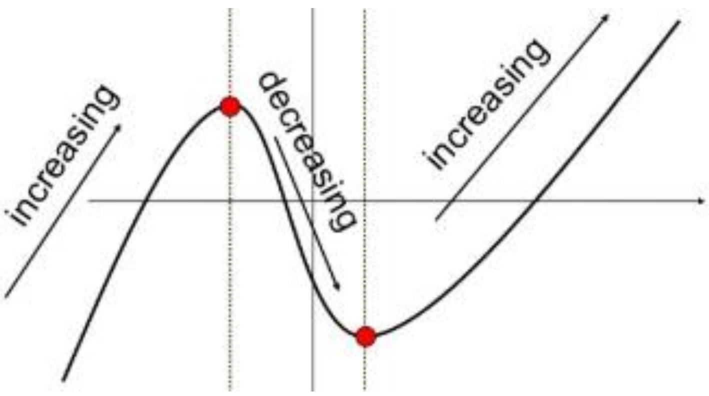
Important Terminology

Domain:	the set of possible x-values	For all functions
Range:	the set of possible y-values	For all functions
Axis of symmetry:	an imaginary line that divides a graph into two mirror images of each other.	See the hyperbola and parabola
Maximum:	the highest possible y-value of a function.	See the parabola
Minimum:	the lowest possible y-value of a function.	See the parabola
Asymptote:	an imaginary line that a graph approaches but never touches.	See the hyperbola and exponential function
Turning point:	The point at which a graph reaches its maximum or minimum value and changes direction.	See the parabola

The concepting of increasing and decreasing in functions: all functions

- The function is INCREASING when the value of y increases as x is increasing from left to right
  - **THE GRAPH GOES UP**

- The function is DECREASING when the value of y decreases as x is increasing from left to right
  - **THE GRAPH GOES DOWN**



Hyperbolic Functions (Hyperbola)

The graph of $y = \frac{a}{x+p} + q$

Standard form of hyperbola

take note that $\boldsymbol{y = \dfrac{2}{x-2} + 1}$

$$\boldsymbol{= \frac{2}{x + (-2)} + 1}$$

The equations of asymptotes are $x = -p$ ***(vertical asymptote)*** and

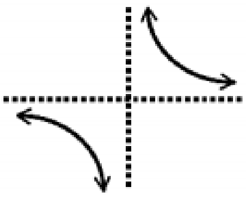
$y = q$ ***(horizontal asymptote)***

Domain: $x \in R, x \neq -p$

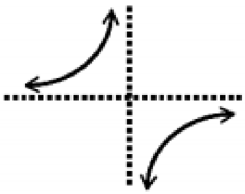
Range: $y \in R, y \neq q$

Shape

If $a > 0$ then the graph decreases for all $x < 0$ or $x > 0$.



If $a < 0$ then the graph increases for all $x < 0$ or $x > 0$.



The equations of the axis of symmetry

The hyperbola has two equations of symmetry

$m = 1$	$m = -1$
$y = x + c$	$y = -x + c$

N.B the equations of the axis of symmetry of the hyperbola passes through the point of intersection of asymptotes $(-p; q)$

In general, for the hyperbola, the equations of the axis of symmetry are given by the following formulae:

$m = 1$	$m = -1$
$y = (x + p) + q$ $\therefore y = x + p + q$	$y = -(x + p) + q$ $\therefore y = -x - p + q$

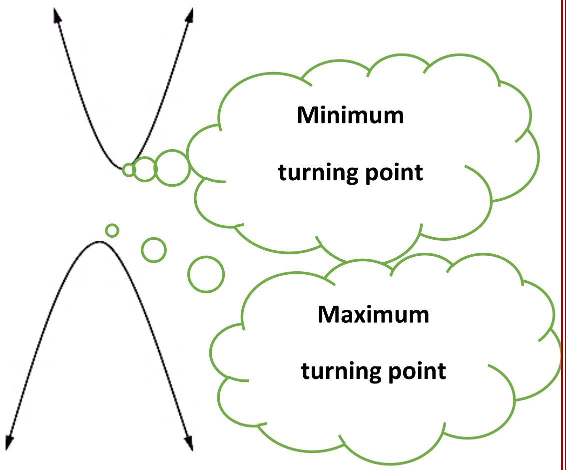
N.B Ensure that the hyperbola is in standard form before applying the formula

Quadratic Function (Parabola)

The graph of $y = a(x+p)^2 + q$

If a is positive, i.e. $a > 0$, then the shape of the graph is ☺.

If a is negative, i.e. $a < 0$, then the shape of the graph is ☹.



The graph has the axis of symmetry at $x = -p$

The graph has the turning point by $(-p\ ;\ q)$

N.B

1. q is the minimum of the parabola when $a > 0$
2. q is the maximum of the parabola when $a < 0$

Domain: $x \in R$

Range: $y \geq q$ 😊 **(WHEN a > 0)**

or

$y \leq q$ 😞 **(WHEN a < 0)**

N.B The parabola changes from increasing to decreasing or decreasing to increasing at the turning point.

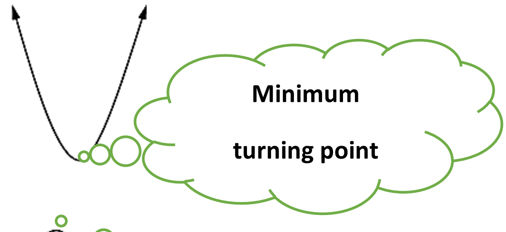
when $a > 0$ 1. *The graph increases for*: $x > -p$ 2. *The graph decrease for*: $x < -p$
when $a < 0$ 1. *The graph increases for*: $x < -p$ 2. *The graph decrease for*: $x > -p$

The quadratic function can also be represented in the form:

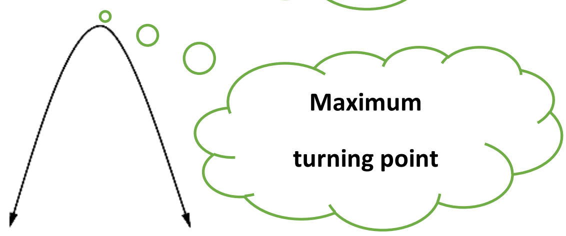
$f(x) = ax^2 + bx + c$

Standard form of parabola

If a is positive, i.e. a > 0, then the shape of the graph is ☺.



If a is negative, i.e. a < 0, then the shape of the graph is ☹.



The graph has the axis of symmetry at $x = -\frac{b}{2a}$

The graph has the turning point by $(-\frac{b}{2a}\ ;\ f(-\frac{b}{2a}))$

Domain: $x \in R$

Range: $y \geq f(-\frac{b}{2a})$ 😊 $\boldsymbol{(WHEN\ a\ >\ 0)}$

or

$y \leq f(-\frac{b}{2a})$ 😔 $\boldsymbol{(WHEN\ a\ <\ 0)}$

N.B

1. $f\left(-\frac{b}{2a}\right)$ **is the minimum of the parabola when** $a > 0$

2. $f\left(-\frac{b}{2a}\right)$ **is the maximum of the parabola when** $a < 0$

Exponential Function

The graph of $y = a.b^{x+p} + q$ where $b > 0$ and $b \neq 1$

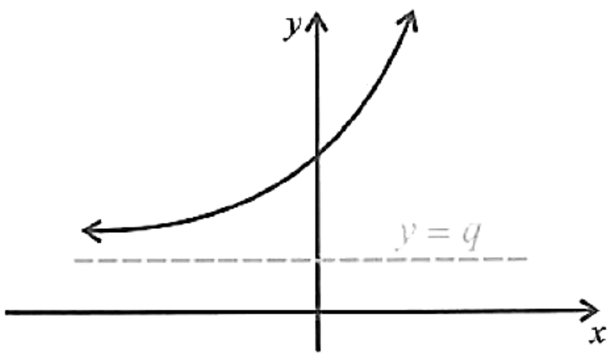
The equation of an asymptote is $y = q$ *(horizontal asymptote)*

Domain: $x \in R$

Range: $y > q$ *[if $a > 0$]* or $y < q$ *[if $a < 0$]*

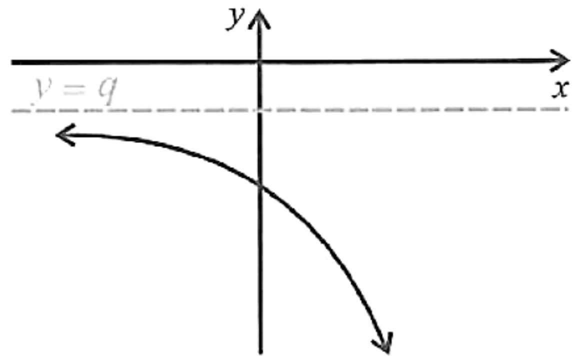
$a > 0$ and $b > 1$

The graph lies above the horizontal asymptote and is an increasing function.



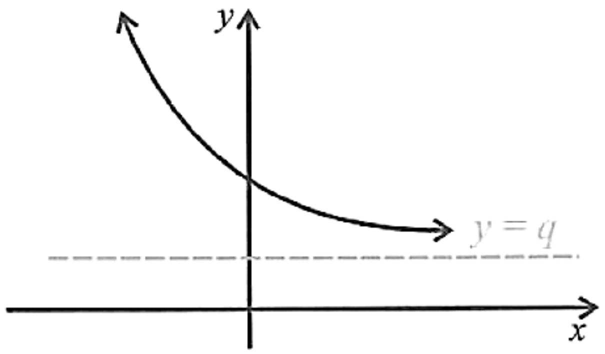
$a < 0$ and $b > 1$

The graph lies below the horizontal asymptote and is a decreasing function.



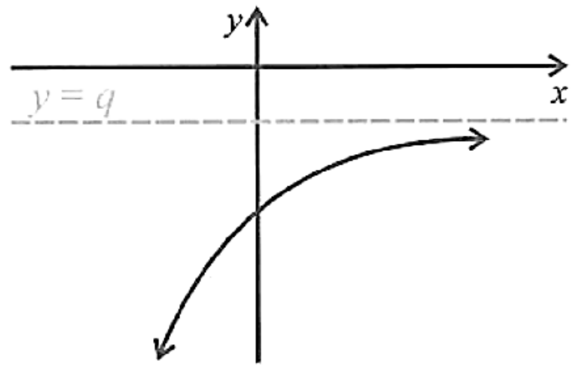
$a > 0$ and $0 < b < 1$

The graph lies above the horizontal asymptote and is a decreasing function.



$a < 0$ and $0 < b < 1$

The graph lies below the horizontal asymptote and is an increasing function.



(Source: MATHS MADE EASY – A comprehensive guide to Grade 12 Mathematics)

Transformation Of Functions

Reflections And Translations

Given $f(x)=\frac{2}{x+1}-3$	**Given $f(x)=2.3^{x-2}+4$**	**Given $f(x)=x^2+5x+6$**
a. The graph of $g(x)$ is obtained by shifting the graph of $f(x)$ 2 units up and 3 units to left. Determine the equation of $g(x)$. **Solution** $g(x)=f(x+3)+2$ $=\frac{2}{x+3+1}-3+2$ $=\frac{2}{x+4}-1$ b. The graph of $h(x)$ is obtained by reflecting the graph of $f(x)$ in the $x-axis$. Determine the equation of $h(x)$. **Solution** $h(x)=-f(x)$ $=-\left(\frac{2}{x+1}-3\right)$ $=-\frac{2}{x+1}+3$ c. The graph of $m(x)$ is obtained by reflecting the graph of $f(x)$ in the $y-axis$. Determine the equation of $m(x)$. **Solution** $m(x)=f(-x)$ $=\frac{2}{-x+1}-3$ $=\frac{2}{-(x-1)}-3$ $=-\frac{2}{x-1}-3$	a. The graph of $g(x)$ is obtained by shifting the graph of $f(x)$ 2 units up and 3 units to left. Determine the equation of $g(x)$. **Solution** $g(x)=f(x+3)+2$ $=2.3^{x+3-2}+4+2$ $=2.3^{x+1}+6$ b. The graph of $h(x)$ is obtained by reflecting the graph of $f(x)$ in the $x-axis$. Determine the equation of $h(x)$. **Solution** $h(x)=-f(x)$ $=-(2.3^{x-2}+4)$ $=-2.3^{x-2}-4$ c. The graph of $m(x)$ is obtained by reflecting the graph of $f(x)$ in the $y-axis$. Determine the equation of $m(x)$. **Solution** $m(x)=f(-x)$ $=2.3^{-x-2}+4$ $=2.3^{-(x+2)}+4$ $=2.\left(\frac{1}{3}\right)^{x+2}+4$	a. The graph of $g(x)$ is obtained by shifting the graph of $f(x)$ 2 units down and 3 units to right. Determine the equation of $g(x)$. **Solution** $g(x)=f(x-3)-2$ $=(x-3)^2+5(x-3)+6-2$ $=x^2-6x+9+5x-15+4$ $=x^2-x-2$ b. The graph of $h(x)$ is obtained by reflecting the graph of $f(x)$ in the $x-axis$. Determine the equation of $h(x)$. **Solution** $h(x)=-f(x)$ $=-(x^2+5x+6)$ $=-x^2-5x-6$ c. The graph of $m(x)$ is obtained by reflecting the graph of $f(x)$ in the $y-axis$. Determine the equation of $m(x)$. **Solution** $m(x)=f(-x)$ $=(-x)^2+5(-x)+6$ $=x^2-5x+6$

Inverse Functions

The concept of a function

A function f, is defined as a relationship between values, where each input value maps to one output value.
In other words, for an equation to be called a function, there can only be one y - value for a particular x-value.

There are two types of functions:

1. **One-to-One Functions**
2. **Many-to-One Functions**

ONE-TO-ONE FUNCTIONS

A one-to-one **function is a function where there is a single y-value for a particular x-value.**

MANY-TO-ONE FUNCTIONS

A function cannot have more than one y value to each x value. However, a function can have more than one x value for a particular y value. **These are known as** many to-one **functions.**

VERTICAL LINE TEST

To test if a graph is a function, use the vertical line **test.** If a vertical line (a line parallel to the y-axis) touches the graph more than once at any point, the graph is not a function. You don't have to draw a line, just hold a ruler parallel to the y-axis and move it along the graph. If the ruler touches the graph more than once for a single x value, anywhere on the graph, then the graph is not a function. In the case of the graph not being a function, it is said to be a relation.

HORIZONTAL LINE TEST

If a graph passes the vertical line test, it is a function. **The** horizontal line **test can be used to determine what type of function the graph represents.**

If a horizontal line (a line parallel to the x-axis) is drawn and moved along the graph and it **touches the graph more than once** at any point, it is a **many-to-one function** (many x-values to a single y-value). Otherwise, it is a one-to-one function.

(Source: MATHS MADE EASY – A comprehensive guide to Grade 12 Mathematics)

Take Note That:

- The inverse of a function takes the y-values (range) of the function to the corresponding x-values (domain) and vice versa. Therefore the x and y values are interchanged.
- The function is reflected along the line $y = x$ to form the inverse.
- The notation for the inverse of a function is f^{-1}.
- N.B The domain of the inverse is the range of the function and the range of the inverse is the domain of the function.
- When the function is increasing, its inverse also increases. When the function decreases, its inverse will also decrease.

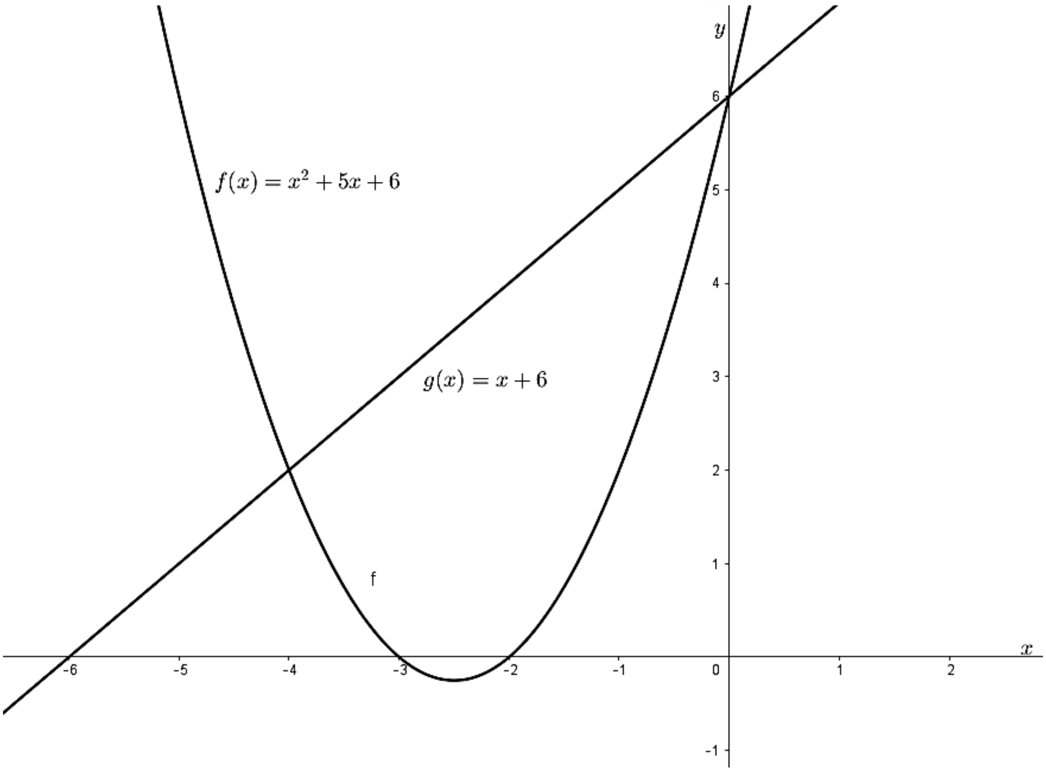
Determine and sketch graphs of the inverses of the functions defined by:

1. $y = ax + q$
2. $y = ax^2$
3. $y = b^x$; $(b > 0, b \neq 1)$

Combination of Functions

Take note of the following when working with combination of functions:

Consider the graphs of f and g below



For $f(x) > 0$ *or* $f(x) < 0$	For $f(x) > g(x)$ *or* $f(x) < g(x)$	For $f(x).g(x) > 0$ *or* $f(x).g(x) < 0$
Focus on the $x - axis$. $f(x) > 0$ means where the graph of $f(x)$ is positive, which will be above the $x - axis$. And $f(x) < 0$ means where the graph of $f(x)$ is negative, which will be below the $x - axis$.	$f(x) > g(x)$ means where the graph of $f(x)$ is above the graph of $g(x)$. And $f(x) < g(x)$ means where the graph of $f(x)$ is below the graph of $g(x)$.	**$f(x).g(x) > 0$ means where the product of $f(x)$ and $g(x)$ is positive.** And **$f(x).g(x) < 0$ means where the product of $f(x)$ and $g(x)$ is negative.**

Examination Guidelines (Functions and Graphs)

Source: Mathematics Examination Guidelines Grade 12, 2021

1. Candidates must be able to use and interpret functional notation. In the teaching process learners must be able to understand how $f(x)$ has been transformed to generate $f(-x)$, $-f(x)$, $f(x+a)$ $f(x)+a$, $af(x)$ and $x = f(y)$ where $a \in R$.
2. Trigonometric functions will ONLY be examined in PAPER 2.

FINANCE, GROWTH AND DECAY

Simple Growth

Formula: $\boldsymbol{A = P(1 + n.i)}$

A = The final amount

P = The initial amount

n = number of years /

i = interest rate in decimals

Compound Growth

Formula: $\boldsymbol{A = P(1 + i)^n}$

A = The final amount

P = The initial amount

n = number of years

i = interest rate in decimals

Simple Decay (Straight line depreciation)

Formula: $\boldsymbol{A = P(1 - n.i)}$

A = The final amount

P = The initial amount

n = number of years

i = interest rate in decimals

N.B the initial amount is bigger than the final amount

Compound Decay (reducing balance depreciation)

Formula: $A = P(1-i)^n$

A = The final amount

P = The initial amount

n = number of years

i = interest rate in decimals

N.B the initial amount is bigger than the final amount

Nominal and Effective Interest rates

Nominal Rate

The rate quoted, and compounding periods are different:

e.g. 10% p.a. compounded quarterly

Effective Rate

The rate quoted, and compounding periods are the same:

e.g. 10% p.a. compounded annually

5% per month compounded monthly

Formula to convert from nominal rate to effective annual rate (and vice versa)

$$1+i_{eff} = \left(1+\frac{i_{nom}}{m}\right)^m$$

i_{eff} = **effective annual rate**

i_{nom} = **nominal rate**

m = **number of compounding periods per year**

Solving for n

$$A = P(1 \pm i)^n$$

$$(1 \pm i)^n = \tfrac{A}{P}$$

$$\log\ (1 \pm i)^n = \log\ \tfrac{A}{P}$$

$$n\ \log(1 \pm i) = \log\ A - \log\ P$$

$$\therefore n = \frac{\log A - \log P}{\log(1 \pm i)}$$

N.B

2,3456 years = 2 years 5 months

How to get 5 months: $[2,3456 - 2] \times 12 = 5\ months$

Future Value Annuity

Formula: $\boldsymbol{F = \frac{x[(1+i)^n - 1]}{i}}$

$\boldsymbol{F} =$ Future value
$x =$ fixed regular payments
$n =$ number of payments
$i =$ interest rate in decimals

When there is "x " immediate payment made, and the last payment is made at the end of the period:

Use the following formula: $\boldsymbol{F = \frac{x\left[(1+i)^{n+1} - 1\right]}{i}}$

When there is an immediate payment made of an amount that is not x , say t , and the last payment is made at the end of the period:

Use the following formula: $\boldsymbol{F = t(1+i)^n + \frac{x[(1+i)^n - 1]}{i}}$

When payments are made at the beginning of each period or when payments are made at the end of each period and the last payment is made, for an example 1 month before the end of the period if interest is compounded monthly:

Use the following formula: $\boldsymbol{F = \frac{x[(1+i)^n - 1]}{i} \times (1+i)^n}$

Sinking Fund

Sinking fund is an amount that is invested to replace something (e.g. Vehicle, Machinery) in future. We use future value annuity to save money in regular intervals for the money to be used in future.

N.B Sinking fund = New price after inflation − Book value

Present Value Annuity

Formula: $\boldsymbol{P = \frac{x[1-(1+i)^{-n}]}{i}}$

$\boldsymbol{P} =$ Present value (loan amount)

$x =$ fixed regular payments

$n =$ number of payments

$i =$ interest rate in decimals

Interest paid

Interest amount paid = All payments made − loan amount

Balance on the loan

Balance = $P(1+i)^n - \frac{x[(1+i)^n-1]}{i}$

OR

Balance = $\boldsymbol{\frac{x[1-(1+i)^{-n}]}{i}}$

$\boldsymbol{n} \rightarrow$ **number of payments left**

Examination Guidelines (Finance, Growth and Decay)

Source: Mathematics Examination Guidelines Grade 12, 2021

1. Understand the difference between nominal and effective interest rates and convert fluently between them for the following compounding periods: monthly, quarterly and half-yearly or semi-annually.
2. With the exception of calculating i in the F_v and P_v formulae, candidates are expected to calculate the value of any of the other variables.
3. Pyramid schemes will NOT be examined in the examination.

DIFFERENTIAL CALCULUS

First Principles

The formula $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is used to find any of the following from **FIRST PRINCIPLES** or by **USE OF THE DEFINITION :**

- The derivative of f at any point.
- The gradient of the tangent to graph f at any point.
- The instantaneous rate of change of a function at any point.
- The gradient of a function at any point.

You must be able to find the derivatives, **FROM FIRST PRINCIPLES** of the following types for exam purpose:

$f(x) = c$, $\quad f(x) = ax^2 + bx + c$, $\quad f(x) = ax^3$, $f(x) = \frac{a}{x}$

Rules of Differentiation

In grade 12 we use one rule, the power rule /constant multiple rule:

$$\boxed{\frac{d}{dx}\left(ax^n\right) = anx^{n-1} \quad for\ n \in \mathrm{R}}$$

…which means differentiating the function (ax^n) with respect to x.

NOTE: The notation we use for the derivative of $y = f(x)$ is

$f'(x)$ or y' or $\frac{dy}{dx}$ or $D_x[f(x)]$.

When we find the derivative of a function, we say we **differentiate** the function.

- ***N.B the derivative of a constant is zero***

if $f(x) = 5$

then,

$f'(x) = 0$

- Step by step example on how to use the rule:

Differentiate $y = -2x^3$

Solution

$$\frac{d(-2x^3)}{dx} = -2 \times 3x^{3-1} = -6x^2$$

Equation of a Tangent

The slope of the tangent line to the graph at a point is equal to the derivative of the function at that point. So, to find the equation of the tangent line to

$f(x)$ at $x = a$, we must:

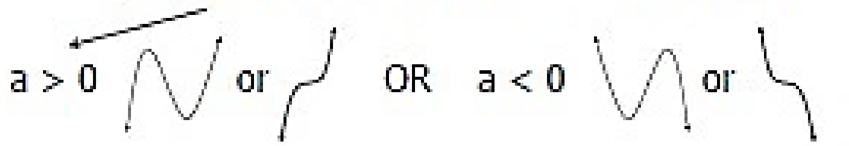
a) Find the derivative $f\,'(x)$
b) Work out the derivative at $x = a \rightarrow$ i.e calculate $f\,'(a)$ to get the gradient of the tangent line.
c) Calculate the y value at $x = a \rightarrow$i.e calculate $f(a)$.
d) The tangent line is a straight line. We can find the equation of a straight line using $y - y_1 = m(x - x_1)$

N.B To calculate the equation of a tangent, you need to the point of contact and the gradient.

Sketching the Cubic Function

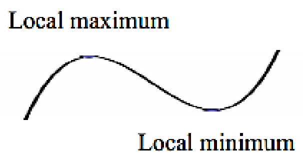
Standard form:

$y = ax^3 + bx^2 + cx + d \longrightarrow$ y-intercept

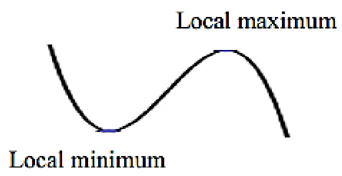


Take note of different terminology used for turning points (stationary points)

If $a > 0$



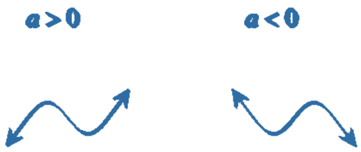
If $a < 0$



Steps to follow when sketching cubic functions:

(N.B: Sometimes, if not most of the time, you will be directed by the question as to where to start or how to start sketching)

1. Shape→

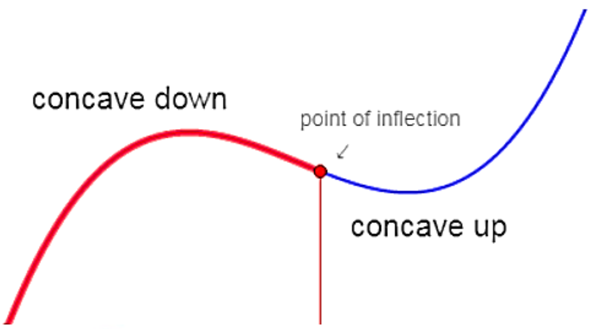


2. Determine the x and $y-intercepts$
   a. x-intercept
      i. Let $y = 0$ and simplify
      ii. Find the factor
      iii. Solve for x
   b. y-intercept
      i. Let $x = 0$ and solve for y

3. Determine the turning points (stationary points)
   a. Find the first derivative, $\boldsymbol{f'(x)}$
   b. Equate first derivative to zero
   c. Solve for x (these are x-values of the turning points)
   d. Substitute the x-values into the original function**,** $\boldsymbol{f(x)}$**,** to find the corresponding y-values)

4. Sketch the cubic function
5. (N.B When asked to find the point of inflection (P.O.I), you must Find the second derivative, $f''(x)$→ Equate the second derivate to zero → Solve for x to get the x-value of P.O.I → Substitute x value into the original function, $f(x)$, to get the corresponding y-value).

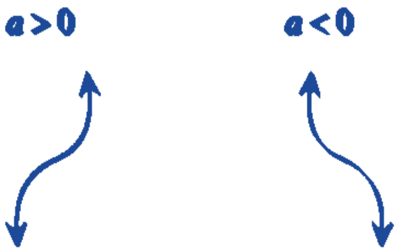
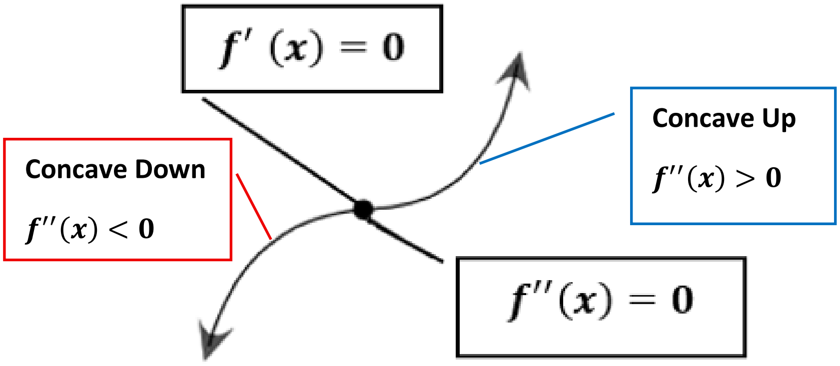
Concavity (Concavity Changes at the P.O.I)



Concave down: $f''(x) < 0$

Concave up: $f''(x) > 0$

The characteristics of the graph of $f(x) = ax^3$

Shape	Stationary Point and P.O.I
	 The point of inflection is the same as the stationary point.

Applications

Optimisation and rate of change, including calculus of motion

Maximum and minimum occur at the turning points, thus when maximizing or minimizing, find the first derivative, equate the derivative to zero and solve for the unknown, x in most cases. The value(s) of x found after solving is/are where the maximum or the minimum occurs. Check the value(s) whether they give maximum or minimum values by substituting into original equation.

Examination Guidelines (Differetial Calculus)

Source: Mathematics Examination Guidelines Grade 12, 2021

1. The following notations for differentiation can be used: $f'(x)$, D_x, $\dfrac{dy}{dx}$ or y'.
2. In respect of cubic functions, candidates are expected to be able to:
   - Determine the equation of a cubic function from a given graph.
   - Discuss the nature of stationary points including local maximum, local minimum and points of inflection.
   - Apply knowledge of transformations on a given function to obtain its image.
3. Candidates are expected to be able to draw and interpret the graph of the derivative of a function.
4. Surface area and volume will be examined in the context of optimisation.
5. Candidates must know the formulae for the surface area and volume of the right prisms. These formulae will NOT be provided on the formula sheet
6. If the optimisation question is based on the surface area and/or volume of the cone, sphere and/or pyramid, a list of the relevant formulae will be provided in that question. Candidates will be expected to select the correct formula from this list.

COUNTING PRINCIPLE AND PROBABILITY

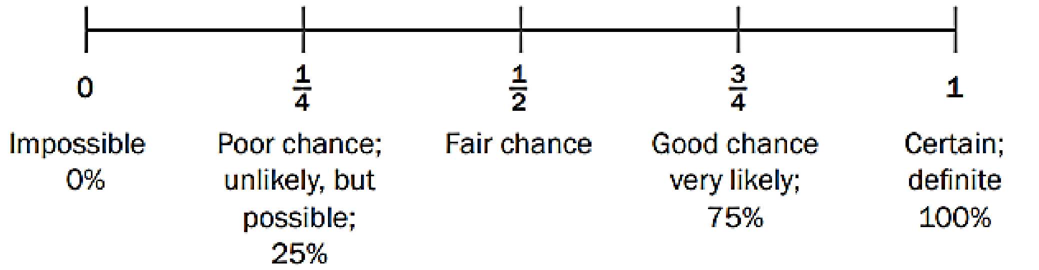
General Probability

Probability refers to the likelihood or chance of an event taking place.

$$\textbf{\textit{The probability of an event}} = \frac{\textbf{\textit{number of favourable outcomes}}}{\textbf{\textit{total number of outcomes}}}$$

This ratio can be expressed as a common fraction, a decimal fraction or a percentage. So a probability of 5 out of 8 can be written as $\frac{5}{8}$ or as 0,625 or as 62,5%.

We can use a **probability scale** to decide what chance there is of an event happening.



Notations that are used are to find probability of an event:

- $P(A)$ means the probability of event A occurring
- $P(A')$ or $P(not\ A)$ means the probability of event A not occurring.
- $P(A\ or\ B) = P(A \cup B)$ means the probability of A or B occurring.

$\cup$ is the symbol for **or**, it is also known as union.

- $P\ (A\ and\ B) = P\ (\ A\ \cap\ B\ )$ means the probability of A and B occurring.

$\cap$ is the symbol for **and**, it is also known as intersection.

The Identity

For any two events A and B:

$$\boldsymbol{P(A\ or\ B)\ =\ P(A)\ +\ P(B)\ -\ P(A\ and\ B)}$$

The Addition Rule for Mutually Exclusive Events

Two events are called mutually exclusive if both events cannot take place at the same time.

When events A and B are mutually exclusive, $\boldsymbol{P(A\ and\ B)\ =\ 0}$

then $\boldsymbol{P(A\ or\ B) = P(A) + P(B)}$

The Complementary Rule:

For two events A and B:

$$\boldsymbol{P(not\ A\ ) = 1 - P(A)}$$

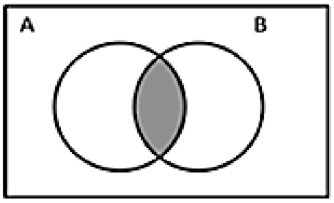
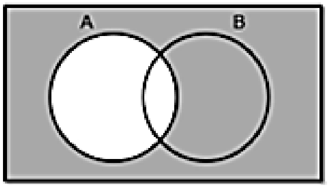
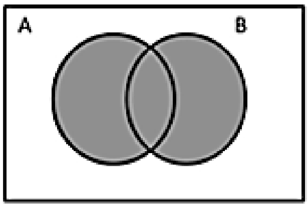
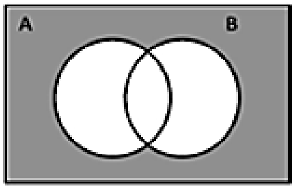
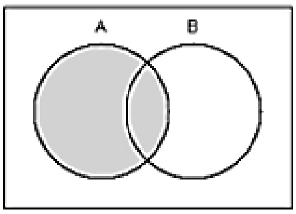
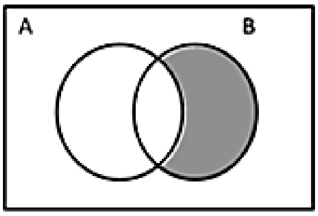
The Product Rule for Independent Events

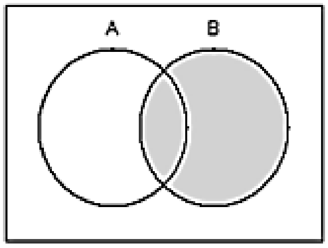
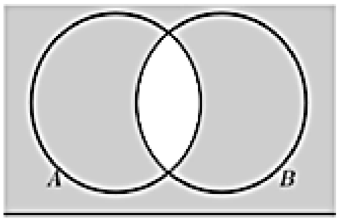
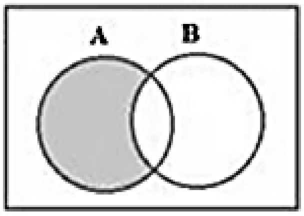
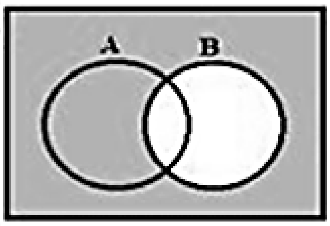
When two events A and B are independent, then

$$\boldsymbol{P(A\ and\ B)\ =\ P(A) \times P(B)}$$

Venn Diagrams

The shaded regions represent the events above the Venn diagrams.

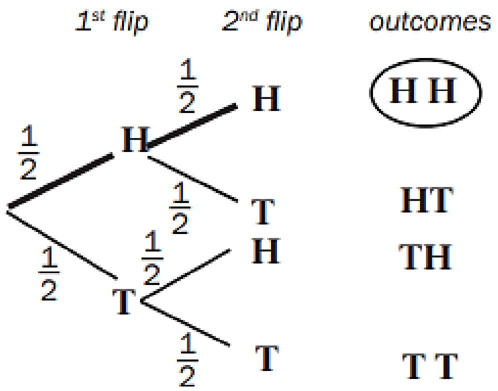
A and B	not A
	
A or B	not (A or B)
	
A	B only
	

B	not (A and B)
	
A only	not B
	

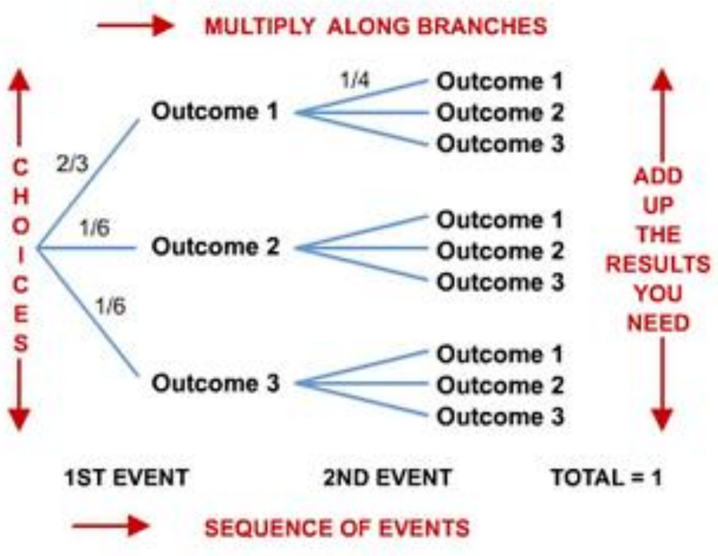
Tree Diagrams

A tree diagram is a picture that helps you to list all possible outcomes of the events. A tree diagram is a tool that is very useful for questions about **replacement and non-replacement**.

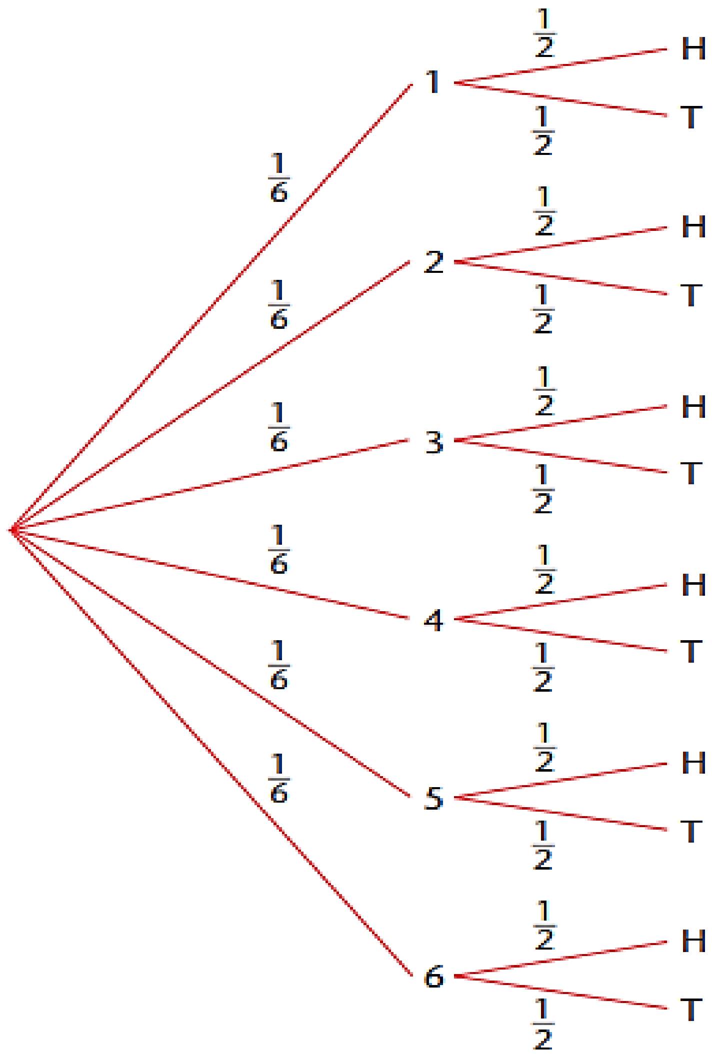
Here is the tree diagram for P(H and H) if you flip a coin twice:



In a tree diagram (once it is complete), to find the probability of an outcome we multiply across the branches. If more than one outcome matches the results required, then addition of the answers gained from the multiplication is used. Remember that each 'clump' of branches should add up to a probability of 1.



When more than one event takes place consecutively or simultaneously, it is useful to represent them as a tree diagram. We represent each event by a column of branches, and the number of branches is determined by the number of possible outcomes for that event. For example, if a die is thrown, there are six possible outcomes, numbers 1 to 6, which we represent by six different line (branches) drawn from the same starting point. If a coin is the tossed (with two possible outcomes , head or tail), we draw the tree diagram as shown below:



Two-Way Contingency Tables

Two-way contingency tables are tools used to organize and display data involving **two categorical variables**. They help learners analyze and calculate probabilities based on real-world or survey data.

	A_1	B_1	**TOTAL**
B_1	p	q	$\boldsymbol{p+q}$
B_2	r	s	$\boldsymbol{r+s}$
TOTAL	$\boldsymbol{p+r}$	$\boldsymbol{q+s}$	$\boldsymbol{p+q+r+s=n}$

Below are the few examples on how to find probabilities using contingency table.

$$P(A_1) = \frac{\boldsymbol{p+r}}{n}$$

$$P(B_2) = \frac{\boldsymbol{r+s}}{n}$$

$$P(A_1 \cap B_2) = \frac{r}{n}$$

Fundamental Counting Principle

- The ***fundamental counting principle*** is a quick method for calculating numbers of outcomes using multiplication.

- The fundamental counting principle states:

  Suppose there are n_1 ways to make a choice, and for each of these there are n_2 ways to make a second choice, and for each of these there are n_3 ways to make a third choice, and so on.
  The product $n_1 \times n_2 \times n_3 \times \ldots \times n_k$ is the number of possible outcomes.

In simple language the fundamental counting principle says:
"If you have several stages of an event, each with a different number of outcomes, then you can find the TOTAL number of outcomes by multiplying the number of outcomes of each stage."

Source: Counting Since 2014 – Pat Tshikane

Steps to Solve Counting Principle Problems

1. Understand the Scenario

- Read the problem carefully.
- Identify what is being counted (e.g., passwords, outfits, arrangements).
- Determine how many **stages** or **choices** are involved.

2. Break the Problem into Stages

- Each stage represents a decision or selection (e.g., choosing a letter, a digit, a color).
- Write down how many options are available at each stage.

3. Check for Restrictions

- Are there any **rules** like:
  - No repetition?
  - Must be even/odd?
  - Must start with a certain letter?
- Adjust the number of choices accordingly.

4. Apply the Counting Principle

- Multiply the number of choices at each stage:
- Total outcomes=Choices at Stage 1×Choices at Stage 2×...

5. Double-Check Your Work

- Make sure you've considered all stages and restrictions.
- Recalculate if needed to confirm your answer.

Source: Counting Since 2014 – Pat Tshikane

Examination Guidelines (Counting Principle and Probability)

Source: Mathematics Examination Guidelines Grade 12, 2021

1. Dependent events are examinable but conditional probabilities are not part of the syllabus.
2. Dependent events in which an object is not replaced are examinable.
3. Questions that require the learner to count the different number of ways that objects may be arranged in a circle and/or the use of combinations are not in the spirit of the curriculum.
4. In respect of word arrangements, letters that are repeated in the word can be treated as the same (indistinguishable) or different (distinguishable). The question will be specific in this regard.

SECTION 2: Paper 1 – Activities

PAPER 1 ACTIVITIES

Algebra, Equations and Inequalities

(May/June 2024)

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 + 5x = 0$ (2)

1.1.2 $4x^2 + 3x - 5 = 0$ (answers correct to TWO decimal places) (3)

1.1.3 $(x-1)^2 - 9 \geq 0$ (4)

1.1.4 $5^{2x} - 5^x = 0$ (4)

1.1.5 $\dfrac{x}{\sqrt{20-x}} = 1$ (5)

1.2 Solve for x and y simultaneously:

$x + y = 9$ and $2x^2 - y^2 = 7$ (5)

1.3 Given: $P = (1-a)$ and $T = (1+a)(1+a^2)(1+a^4)...(1+a^{512})$

Determine the value of $P \times T$ in terms of a. (3)

[26]

(May/June 2023)

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 7x + 12 = 0$ (3)

1.1.2 $x(3x+5) = 1$ (correct to TWO decimal places) (4)

1.1.3 $x^2 < -2x + 15$ (4)

1.1.4 $\sqrt{2(1-x)} = x - 1$ (4)

1.2 Solve for x and y simultaneously:

$3^{x+y} = 27$ and $x^2 + y^2 = 17$ (6)

1.3 Determine, **without the use of a calculator**, the value of:

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \frac{1}{\sqrt{99}+\sqrt{100}}$$ (3)

[24]

(May/June 2022)

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 2x - 15 = 0$ (3)

1.1.2 $5x^2 - x - 9 = 0$ (Leave your answer correct to TWO decimal places.) (3)

1.1.3 $x^2 \leq 3x$ (4)

1.2 Given: $a + \frac{64}{a} = 16$

1.2.1 Solve for a. (3)

1.2.2 Hence, solve for x: $2^x + 2^{6-x} = 16$ (3)

1.3 **Without using a calculator**, calculate the value of $\sqrt{\frac{2^{1002} + 2^{1006}}{17(2)^{998}}}$ (4)

1.4 Solve for x and y simultaneously:

$2x - y = 2$ and $\frac{1}{x} - 3y = 1$ (6)

[26]

Patterns and Sequences

(May/June 2024)

QUESTION 2

2.1 Consider the geometric series: $4+2+1+\frac{1}{2}+...$

2.1.1 Does this series converge? Justify your answer. (2)

2.1.2 Calculate S_∞. (2)

2.2 Given: $\sum_{p=k}^{10} 3^{p-1} = 29\ 520$. Calculate the value of k. (5)

[9]

QUESTION 3

3.1 Consider the quadratic number pattern: 3 ; 7 ; 12 ; …

3.1.1 Show that the general term of this number pattern is given by $T_n = \frac{1}{2}n^2 + \frac{5}{2}n$. (3)

3.1.2 What number must be added to T_{n-1} so that $T_n = 13\ 527$? (4)

3.2 Given an arithmetic sequence with $T_1 = 8$ and $T_2 = 11$.

3.2.1 Calculate the value of n if $T_n = 41$. (3)

3.2.2 A new arithmetic sequence P is formed using the term position and the term value of the given arithmetic sequence.
For the new sequence, $P_8 = 1$, $P_{11} = 2$ and so forth.

(a) Write down the value of P_{41}. (1)

(b) Calculate the value of the first term of the new arithmetic sequence. (4)

[15]

(May/June 2023)

QUESTION 2

2.1 Given the geometric series: $\frac{1}{5}+\frac{1}{15}+\frac{1}{45}+\ldots$

2.1.1 Is this a convergent geometric series? Justify your answer with the necessary calculations. (2)

2.1.2 Calculate the sum to infinity of this series. (2)

2.2 An arithmetic and a geometric sequence are combined to form the pattern, which is given by: $P_n = x\,;\frac{1}{3}\,;2x\,;\frac{1}{9}\,;3x\,;\frac{1}{27}\,;\ldots$

2.2.1 Write down the next TWO terms of the pattern. (2)

2.2.2 Determine the general term (T_n) for the odd terms of this pattern. Write down your answer in terms of x. (2)

2.2.3 Calculate the value of P_{26}. (3)

2.2.4 If $\sum_{n=1}^{21} P_n = 33{,}5$, determine the value of x. (6)

[17]

QUESTION 3

A quadratic sequence has the following properties:

- The second difference is 10.
- The first two terms are equal, i.e. $T_1 = T_2$.
- $T_1 + T_2 + T_3 = 28$

3.1 Show that the general term of the sequence is $T_n = 5n^2 - 15n + 16$. (6)

3.2 Is 216 a term in this sequence? Justify your answer with the necessary calculations. (3)

[9]

(May/June 2022)

QUESTION 2

2.1 The first term of an arithmetic sequence is -1 and the 7^{th} term is 35.

Determine:

2.1.1 The common difference of the sequence (2)

2.1.2 The number of terms in the sequence if the last term of the sequence is 473 (3)

2.1.3 The sum of the first 40 terms in this sequence (2)

2.2 $75\ ;\ 53\ ;\ 35\ ;\ 21\ ;\ ...$ is a quadratic number pattern.

2.2.1 Write down the FIFTH term of the number pattern. (1)

2.2.2 Determine the n^{th} term of the number pattern. (4)

2.2.3 Determine the maximum value of the following number pattern:

$$-15\ ;-\frac{53}{5}\ ;-7\ ;-\frac{21}{5}\ ;\ldots$$

(4)

[16]

QUESTION 3

3.1 Consider the following geometric sequence: $1\,024\ ;\ 256\ ;\ 64\ ;\ \ldots$

Calculate:

3.1.1 The 10^{th} term of the sequence (2)

3.1.2 $\sum_{p=0}^{8} 256\left(4^{1-p}\right)$ (4)

3.2 The first two terms of a geometric sequence are:

$$-t^2-6t-9 \text{ and } \frac{t^3+9t^2+27t+27}{2}$$

Determine the values of t for which the sequence will converge. (5)

[11]

Functions and Graphs

(May/June2021)

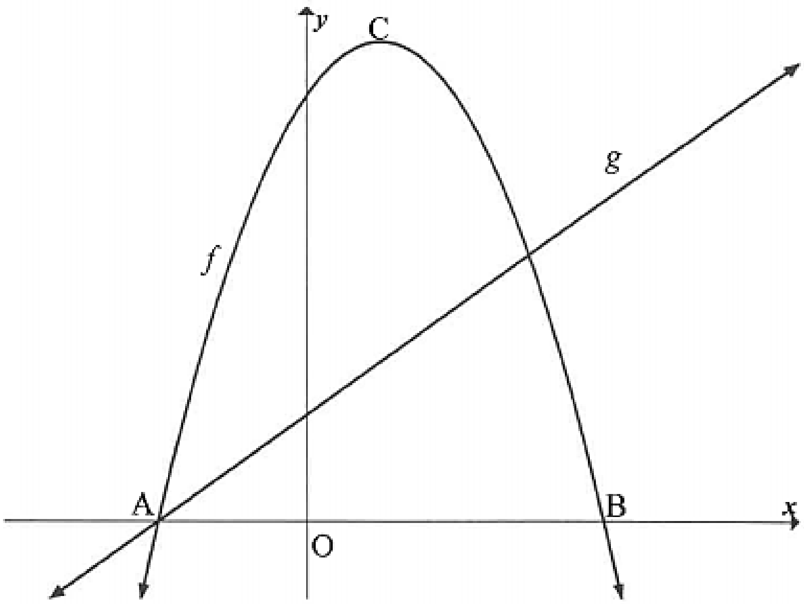
QUESTION 4

The lines $y = x + 1$ and $y = -x - 7$ are the axes of symmetry of the function $f(x) = \frac{-2}{x+p} + q$.

4.1	Show that $p = 4$ and $q = -3$.	(4)
4.2	Calculate the x-intercept of f.	(2)
4.3	Sketch the graph of f. Clearly label ALL intercepts with the axes and the asymptotes.	(4)
		[10]

QUESTION 5

Sketched below are the graphs of $f(x) = -2x^2 + 4x + 16$ and $g(x) = 2x + 4$.
A and B are the x-intercepts of f. C is the turning point of f.



5.1 Calculate the coordinates of A and B. (3)

5.2 Determine the coordinates of C, the turning point of f. (2)

5.3 Write down the range of f. (1)

5.4 The graph of $h(x) = f(x + p) + q$ has a maximum value of 15 at $x = 2$. Determine the values of p and q. (3)

5.5 Determine the equation of g^{-1}, the inverse of g, in the form $y = \ldots$ (2)

5.6 For which value(s) of x will $g^{-1}(x).g(x) = 0$? (2)

5.7 If $p(x) = f(x) + k$, determine the value(s) of k for which p and g will NOT intersect. (5)

[18]

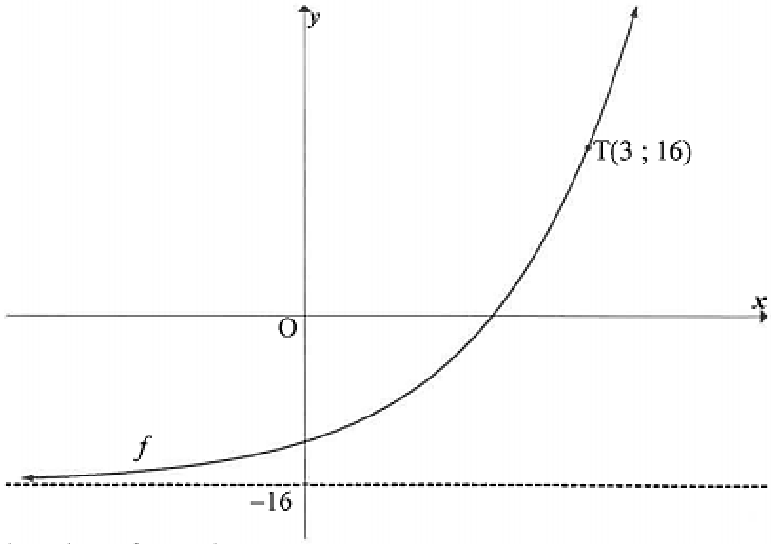
QUESTION 6

6.1 Given: $g(x) = 3^x$

6.1.1 Write down the equation of g^{-1} in the form $y = \ldots$ (2)

6.1.2 Point P(6 ; 11) lies on $h(x) = 3^{x-4} + 2$. The graph of h is translated to form g. Write down the coordinates of the image of P on g. (2)

6.2 Sketched is the graph of $f(x) = 2^{x+p} + q$. T(3 ; 16) is a point on f and the asymptote of f is $y = -16$.



Determine the values of p and q. (4)

[8]

(May/June 2019)

QUESTION 4

Given the exponential function: $g(x) = \left(\frac{1}{2}\right)^x$

4.1 Write down the range of g. (1)

4.2 Determine the equation of g^{-1} in the form $y = \ldots$ (2)

4.3 Is g^{-1} a function? Justify your answer. (2)

4.4 The point $M(a\,;2)$ lies on g^{-1}.

4.4.1 Calculate the value of a. (2)

4.4.2 M', the image of M, lies on g. Write down the coordinates of M'. (1)

4.5 If $h(x) = g(x+3) + 2$, write down the coordinates of the image of M' on h. (3)

[11]

QUESTION 5

5.1 Given: $f(x)=\dfrac{1}{x+2}+3$

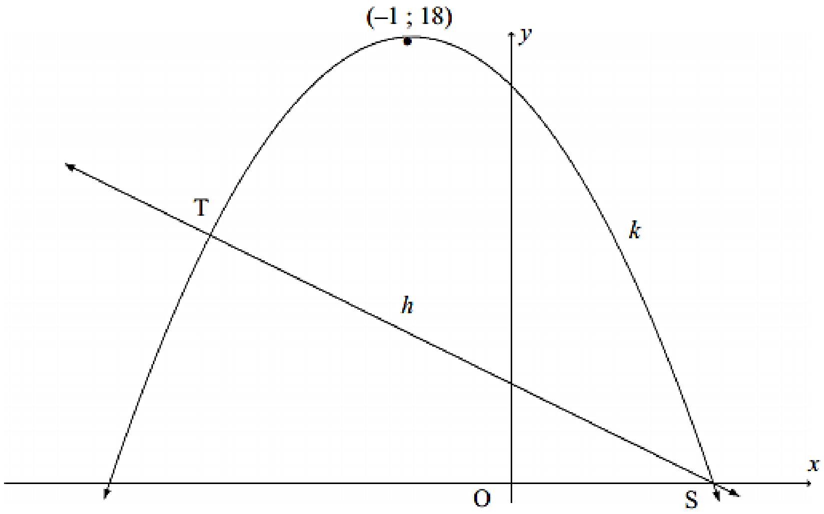
5.1.1 Determine the equations of the asymptotes of f. (2)

5.1.2 Write down the y-intercept of f. (1)

5.1.3 Calculate the x-intercept of f. (2)

5.1.4 Sketch the graph of f. Clearly label ALL intercepts with the axes and any asymptotes. (3)

5.2 Sketched below are the graphs of $k(x)=ax^2+bx+c$ and $h(x)=-2x+4$. Graph k has a turning point at $(-1\ ;\ 18)$. S is the x-intercept of h and k. Graphs h and k also intersect at T.



5.2.1 Calculate the coordinates of S. (2)

5.2.2 Determine the equation of k in the form $y=a(x+p)^2+q$ (3)

5.2.3 If $k(x)=-2x^2-4x+16$, determine the coordinates of T. (5)

5.2.4 Determine the value(s) of x for which $k(x)<h(x)$. (2)

5.2.5 It is further given that k is the graph of $g'(x)$.

(a) For which values of x will the graph of g be concave up? (2)

(b) Sketch the graph of g, showing clearly the x-values of the turning points and the point of inflection. (3)

[25]

(May/June 2024)

QUESTION 4

Given: $g(x)=\dfrac{1}{x-1}+2$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Draw a graph of g, indicating any intercepts with the axes and asymptotes. (4)

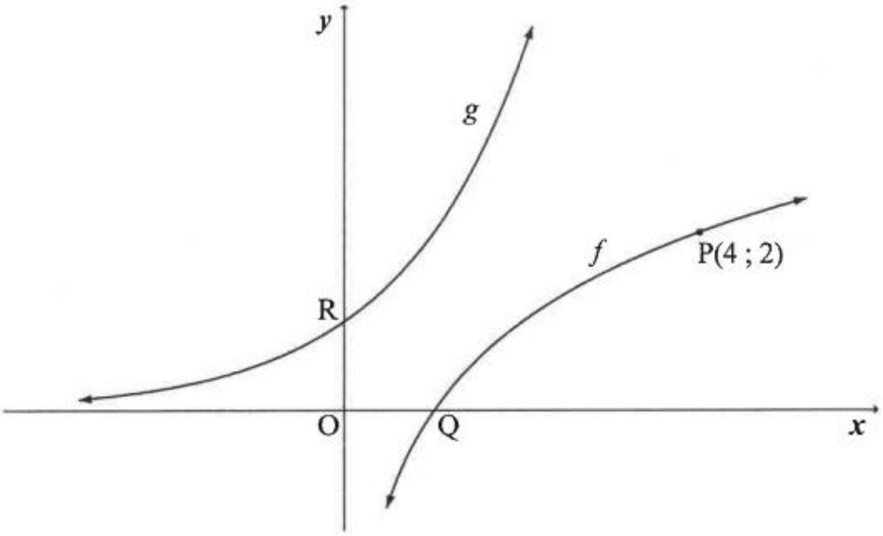
4.3 Determine the values of x where $g(x)>0$. (2)

4.4 Determine the equation of the axis of symmetry of g which has a negative gradient. (2)

[10]

QUESTION 5

In the diagram, the graphs of $f(x)=\log_a x$ and g are drawn. Graph g is the reflection of f in the line $y=x$. Graph f passes through the point P(4 ; 2). Q is the x-intercept of f and R is the y-intercept of g.



5.1 Write down the coordinates of $P^{/}$, the image of P on g. (2)

5.2 Show that $a=2$. (2)

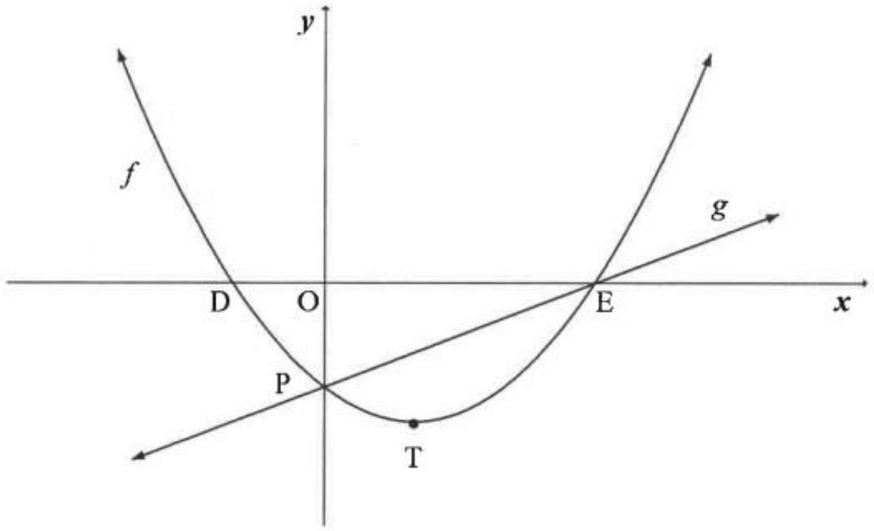
5.3 Write down the equation of g in the form $y=\ldots$ (1)

5.4 T is a point on f in the first quadrant where TR is parallel to the x-axis. Calculate the area of $\Delta RTP^{/}$. (4)

[9]

QUESTION 6

The graphs of $f(x)=x^2-2x-3$ and $g(x)=mx+c$ are drawn below. D and E are the x-intercepts and P is the y-intercept of f. The turning point of f is T$(1\ ;-4)$. The graphs of f and g intersect at P and E.



6.1 Write down the range of f. (1)

6.2 Calculate the coordinates of D and E. (3)

6.3 Determine the equation of g. (2)

6.4 Write down the values of x for which $f(x)-g(x)>0$. (2)

6.5 Determine the maximum vertical distance between h and g if $h(x)=-f(x)$ for $x\in[-2\ ;3]$. (5)

6.6 Given: $k(x)=g(x)-n$.

Determine n if k is a tangent to f. (5)

[18]

(May/June 2023)

QUESTION 4

4.1 Given the function $p(x)=\left(\frac{1}{3}\right)^x$.

4.1.1 Is p an increasing or decreasing function? (1)

4.1.2 Determine p^{-1}, the inverse of p, in the form $y = \ldots$ (2)

4.1.3 Write down the domain of p^{-1}. (1)

4.1.4 Write down the equation of the asymptote of $p(x)-5$. (1)

4.2 Given: $f(x)=\dfrac{4}{x-1}+2$

4.2.1 Write down the equations of the asymptotes of f. (2)

4.2.2 Calculate the x-intercept of f. (2)

4.2.3 Sketch the graph of f, label all asymptotes and indicate the intercepts with the axes. (4)

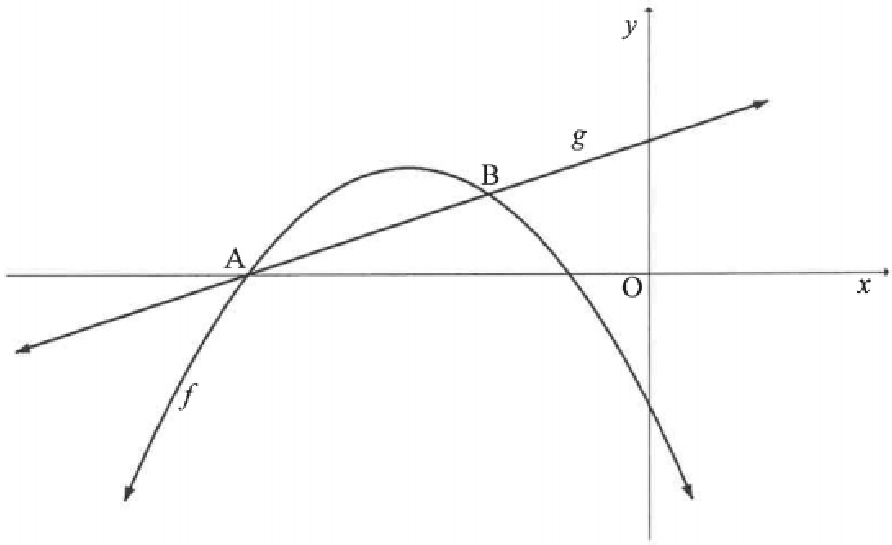
4.2.4 Use your graph to determine the values of x for which $\dfrac{4}{x-1}\geq -2$. (2)

4.2.5 Determine the equation of the axis of symmetry of $f(x-2)$, that has a negative gradient. (3)

[18]

QUESTION 5

The graphs of the functions $f(x) = -(x+3)^2 + 4$ and $g(x) = x + 5$ are drawn below.
The graphs intersect at A and B.



5.1 Write down the coordinates of the turning point of f. (2)

5.2 Write down the range of f. (1)

5.3 Show that the x-coordinates of A and B are -5 and -2 respectively. (4)

5.4 Hence, determine the values of c for which the equation $-(x+c+3)^2 + 4 = (x+c) + 5$ has ONE negative and ONE positive root. (2)

5.5 The maximum distance between f and g in the interval $x_A < x < x_B$ is k.
If $h(x) = g(x) + k$, determine the equation of h in the form $h(x) = \ldots$ (5)

[14]

(May/June 2022)

QUESTION 4

The graph of $g(x) = a\left(\frac{1}{3}\right)^x + 7$ passes through point E(–2 ; 10).

4.1 Calculate the value of a. (3)

4.2 Calculate the coordinates of the y-intercept of g. (2)

4.3 Consider: $h(x) = \left(\frac{1}{3}\right)^x$

4.3.1 Describe the translation from g to h. (2)

4.3.2 Determine the equation of the inverse of h, in the form $y = \ldots$ (2)
[9]

QUESTION 5

Consider: $g(x) = \dfrac{a}{x+p} + q$

The following information of g is given:

- Domain: $x \in \mathbb{R};\ x \neq -2$
- x-intercept at $K(1\ ;\ 0)$
- y-intercept at $N\left(0\ ;\ -\frac{1}{2}\right)$

5.1 Show that the equation of g is given by: $g(x) = \dfrac{-3}{x+2} + 1$ (6)

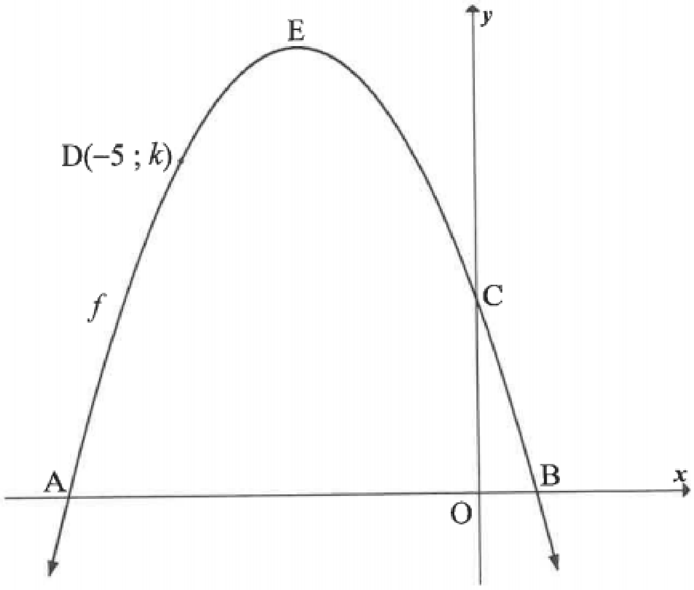
5.2 Write down the range of g. (1)

5.3 Determine the equation of h, the axis of symmetry of g, in the form $y = mx + c$, where $m > 0$. (3)

5.4 Write down the coordinates of $K^{/}$, the image of K reflected over h. (2)
[12]

QUESTION 6

The sketch below shows the graph of $f(x) = -x^2 - 6x + 7$.
C is the y-intercept of f.
A and B are the x-intercepts of f.
D$(-5\ ;\ k)$ is a point on f.



6.1 Calculate the coordinates of E, the turning point of f. (3)

6.2 Write down the value of k. (1)

6.3 Determine the equation of the straight line passing through C and D. (4)

6.4 A tangent, parallel to CD, touches f at P. Determine the coordinates of P. (4)

6.5 For which values of x will $f(x) - 12 > 0$? (2)
[14]

Finance, Growth and Decay

(May/June 2024)

QUESTION 7

7.1 Six years ago, Thabo bought a phone for R13 000. The value of the phone depreciated annually according to the reducing-balance method. The value of the phone is now R8 337,75. Calculate the annual rate of depreciation. (3)

7.2 Eric and Thandi need to save R80 000 each to go on a holiday at the end of December 2027.

- Thandi decides that she will start saving at the end of January 2025. She will make 36 monthly deposits into a savings account that pays interest at 8,6% p.a., compounded monthly. The deposit will be made at the end of each month.
- Eric calculates that if he makes 48 deposits of R1 402,31, starting at the end of January 2024, he will have enough money to go on holiday. He will make his deposits into a savings account at the end of each month. The savings account pays interest at 8,6% p.a., compounded monthly.

Calculate the difference between the total amount that Eric and Thandi will deposit into their respective savings accounts over the given period. (4)

7.3 Lesibana was granted a loan of R225 000. The rate of interest for the loan is 9% p.a., compounded monthly. Lesibana will make monthly payments of R5 500, starting exactly four months after the loan was granted. How many payments will Lesibana make to settle the loan? (6)

[13]

(May/June 2023)

QUESTION 6

6.1 A company bought a photocopier for R150 000 on 1 July 2022. They will use the old photocopier as a trade-in when they replace it with a similar new photocopier in 5 years' time on 30 June 2027.

6.1.1 The average rate of inflation over the next 5 years will be 6,5% p.a. Determine the price of a similar new photocopier in 5 years' time. (2)

6.1.2 Calculate the trade-in value of the old photocopier after 5 years, if it depreciates at a rate of 9% p.a. on a straight-line method. (2)

6.1.3 The company set up a sinking fund to cover the replacement cost of the new photocopier. The fund earns interest at the rate of 7,85% p.a., compounded monthly. The company made its first monthly deposit on 31 July 2022 and will continue to do so until 31 May 2027, one month prior to the new photocopier being bought. How much should be deposited at the end of each month so that the company will be able to buy the new photocopier? (4)

6.2 Today, Andrew borrowed R200 000 from a bank. The bank charges interest at 5,25% p.a., compounded quarterly. Andrew will make repayments of R6 000 at the end of every 3 months. His first repayment will be made in 3 months from now. How long, in years, will it take Andrew to settle the loan? (5)

[13]

(May/June 2022)

QUESTION 7

7.1 How many years will it take for an investment to double in value, if it earns interest at a rate of 8,5% p.a., compounded quarterly? (4)

7.2 A company purchased machinery for R500 000. After 5 years, the machinery was sold for R180 000 and new machinery was bought.

7.2.1 Calculate the rate of depreciation of the old machinery over the 5 years, using the reducing-balance method. (4)

7.2.2 The rate of inflation for the cost of the new machinery is 6,3% p.a. over the 5 years. What will the new machinery cost at the end of 5 years? (2)

7.2.3 The company set up a sinking fund and made the first payment into this fund on the day the old machinery was bought. The last payment was made three months before the new machinery was purchased at the end of the 5 years. The interest earned on the sinking fund was 10,25% p.a., compounded monthly. The money from the sinking fund and the R180 000 from the sale of the old machinery was used to pay for the new machinery.

Calculate the monthly payment into the sinking fund. (5)

[15]

Differential Calculus

(May/June 2021)

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 3x^2$. (5)

8.2 Determine:

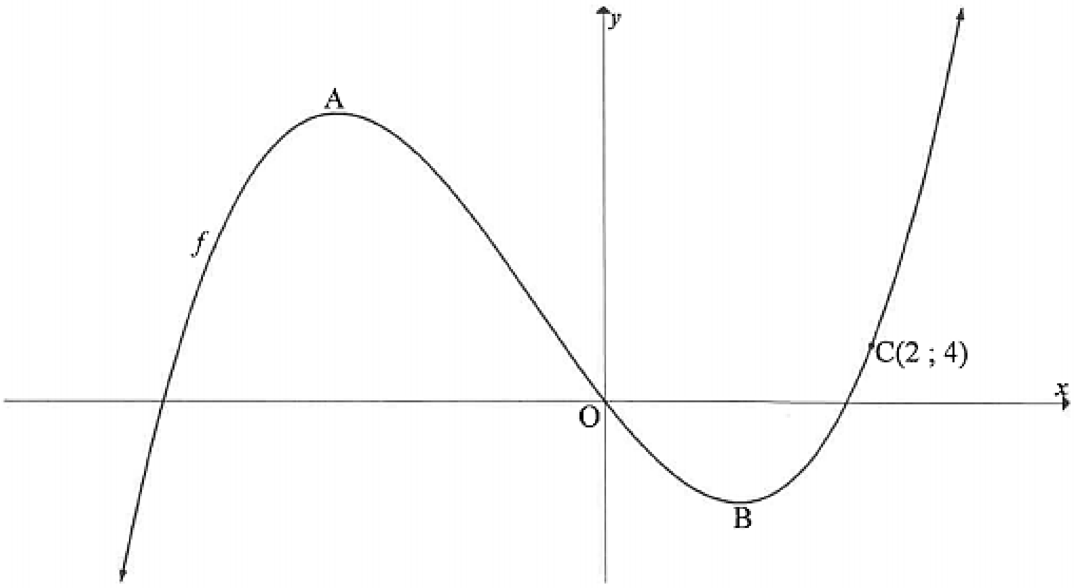
8.2.1 $f'(x)$ if $f(x) = x^2 - 3 + \frac{9}{x^2}$ (3)

8.2.2 $g'(x)$ if $g(x) = (\sqrt{x} + 3)(\sqrt{x} - 1)$ (4)

[12]

QUESTION 9

The graph of $f(x) = 2x^3 + 3x^2 - 12x$ is sketched below.
A and B are the turning points of f. C(2 ; 4) is a point on f.



9.1 Determine the coordinates of A and B. (5)

9.2 For which values of x will f be concave up? (3)

9.3 Determine the equation of the tangent to f at C(2 ; 4). (3)

[11]

QUESTION 10

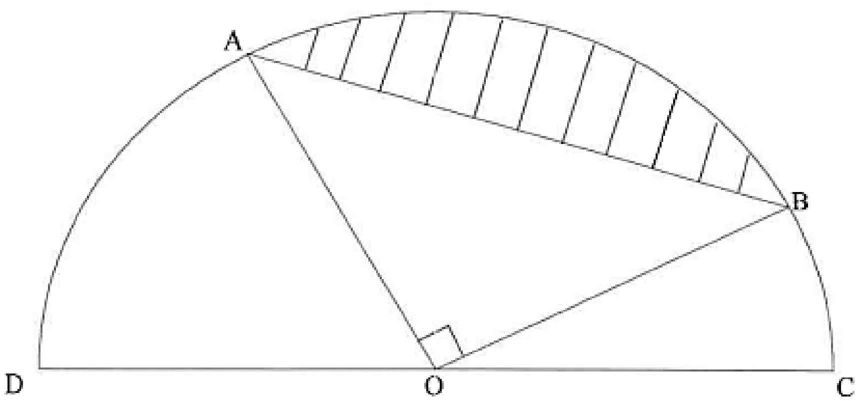
10.1 The graph of $f(x) = ax^3 + bx^2 + cx + d$ has two turning points.

The following information about f is also given:

- $f(2) = 0$
- The x-axis is a tangent to the graph of f at $x = -1$
- $f'(1) = 0$
- $f'\left(\frac{1}{2}\right) > 0$

Without calculating the equation of f, use this information to draw a sketch graph of f, only indicating the x-coordinates of the x-intercepts and turning points. (4)

10.2 O is the centre of a semicircle passing through A, B, C and D. The radius of the semicircle is $(x - x^2)$ units for $0 < x < 1$. ΔAOB is right-angled at O.



10.2.1 Show that the area of the shaded part is given by:

$$\text{Area} = \left(\frac{\pi - 2}{4}\right)\left(x^4 - 2x^3 + x^2\right)$$

(5)

10.2.2 Determine the value of x for which the shaded area will be a maximum. (4)

[13]

(May/June 2019)

QUESTION 7

7.1 Given $f(x) = x^2 + 2$.

Determine $f'(x)$ from first principles. (4)

7.2 Determine $\frac{dy}{dx}$ if:

7.2.1 $y = 4x^3 + \frac{2}{x}$ (3)

7.2.2 $y = 4.\sqrt[3]{x} + (3x^3)^2$ (4)

7.3 If g is a linear function with $g(1) = 5$ and $g'(3) = 2$, determine the equation of g in the form $y = \ldots..$ (3)

[14]

QUESTION 8

A cubic function $h(x) = -2x^3 + bx^2 + cx + d$ cuts the x-axis at $(-3\ ;\ 0)$; $\left(-\frac{3}{2}\ ;\ 0\right)$ and $(1\ ;\ 0)$.

8.1 Show that $h(x) = -2x^3 - 7x^2 + 9$. (3)

8.2 Calculate the x-coordinates of the turning points of h. (3)

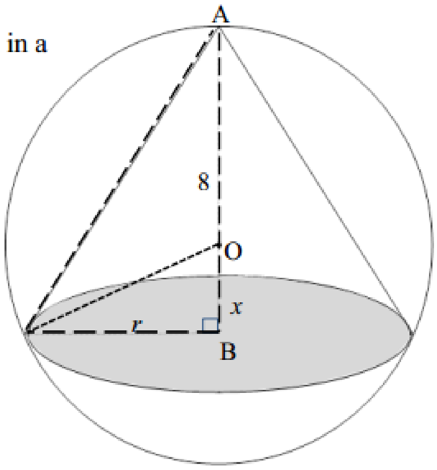
8.3 Determine the value(s) of x for which h will be decreasing. (2)

8.4 For which value(s) of x will there be a tangent to the curve of h that is parallel to the line $y - 4x = 7$. (4)

[12]

QUESTION 9

A cone with radius r cm and height AB is inscribed in a sphere with centre O and a radius of 8 cm. $OB = x$.



Volume of sphere $= \frac{4}{3}\pi r^3$
Volume of cone $= \frac{1}{3}\pi r^2 h$

9.1 Calculate the volume of the sphere. (1)

9.2 Show that $r^2 = 64 - x^2$. (1)

9.3 Determine the ratio between the largest volume of this cone and the volume of the sphere. (7)

[9]

(May/June 2024)

QUESTION 8

8.1 Determine $f^{/}(x)$ from first principles if $f(x)=\dfrac{1}{x}$. (5)

8.2 Determine:

8.2.1 $\dfrac{d}{dx}\left(\sqrt{4x^6}+\sqrt{2}.x^2\right)$ (3)

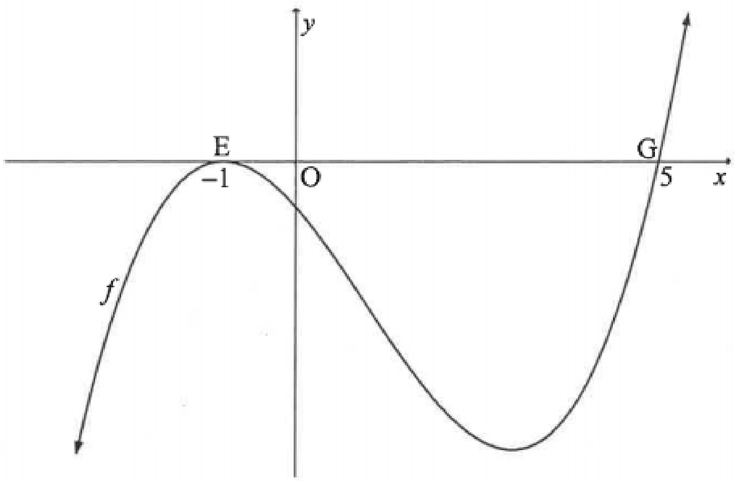
8.2.2 $g^{/}(x)$ if $g(x)=\dfrac{3x^4-4x^2+6}{x^2}$ (3)

8.3 The equation of the tangent to $f(x)=3x^2+bx+c$ at $x=1$ is given by $y=9x-9$. Determine the values of b and c. (4)

[15]

QUESTION 9

The graph of $f(x)=ax^3+bx^2+cx-5$ is drawn below. E(– 1 ; 0) and G(5 ; 0) are the x-intercepts of f.



9.1 Show that $a=1$, $b=-3$ and $c=-9$. (3)

9.2 Calculate the value of x for which f has a local minimum value. (4)

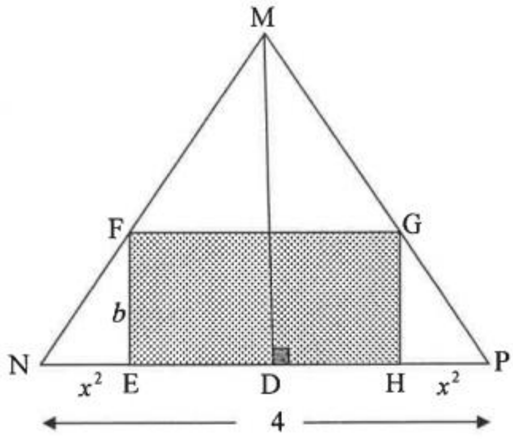
9.3 Use the graph to determine the values of x for which $f^{//}(x).f(x)>0$. (3)

9.4 For which values of t will the graph of $p(x)=f(x)+t$ have two distinct positive roots and one negative root? (3)

[13]

QUESTION 10

EHGF is a rectangle. HE is produced x^2 cm to N and EH is produced x^2 cm to P. NF produced intersects PG produced at M to form an isosceles triangle MNP with NM = MP. D lies on NP where MD ⊥ NP. NP = 4 cm and MD = 3 cm.



10.1	Show that the area of EFGH is given by $A(x) = 6x^2 - 3x^4$.	(4)
10.2	Calculate the maximum area of rectangle EFGH.	(4) **[8]**

(May/June 2023)

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = -2x^2 - 1$. (5)

7.2 Determine:

7.2.1 $f'(x)$, if it is given that $f(x) = -2x^3 + 3x^2$ (2)

7.2.2 $\frac{dy}{dx}$ if $y = 2x + \frac{1}{\sqrt{4x}}$ (4)

7.3 The graph $y = f'(x)$ has a minimum turning point at $(1\,;-3)$.
Determine the values of x for which f is concave down. (2)
[13]

QUESTION 8

Given: $f(x)=x^3+4x^2-7x-10$

8.1 Write down the y-intercept of f. (1)

8.2 Show that 2 is a root of the equation $f(x)=0$. (2)

8.3 Hence, factorise $f(x)$ completely. (3)

8.4 If it is further given that the coordinates of the turning points are approximately at (0,7 ; – 12,6) and (– 3,4 ; 20,8), draw a sketch graph of f and label all intercepts and turning points. (3)

8.5 Use your graph to determine the values of x for which:

8.5.1 $f'(x)<0$ (2)

8.5.2 The gradient of a tangent to f will be a minimum (2)

8.5.3 $f'(x).f''(x)\leq 0$ (3)

[16]

QUESTION 9

A wire, 12 metres long, is cut into two pieces. One part is bent to form an equilateral triangle and the other a square. A side of the triangle has a length of $2x$ metres.

9.1 Write down the length of a side of the square in terms of x. (2)

9.2 If this square is now used as the base of a rectangular prism with a height of $4x$ metres, determine the maximum volume of the rectangular prism. (7)

[9]

(May/June 2022)

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x)=-x^2$. (5)

8.2 Determine:

8.2.1 $f'(x)$, if it is given that $f(x)=4x^3-5x^2$ (2)

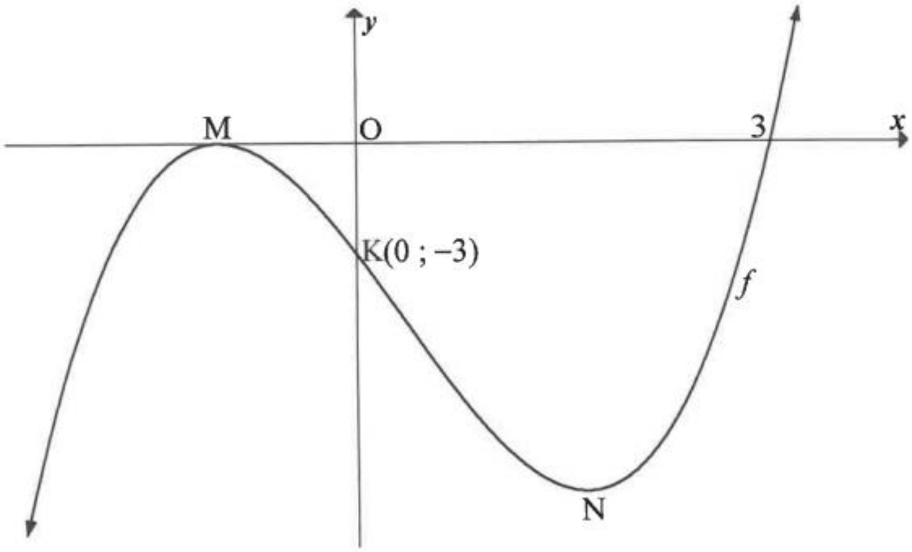
8.2.2 $D_x\left[\dfrac{-6\sqrt[3]{x}+2}{x^4}\right]$ (4)

[11]

QUESTION 9

Sketched below is the graph of $f(x) = x^3 + ax^2 + bx + c$.
The x-intercepts of f are at $(3\ ;\ 0)$ and M, where M lies on the negative x-axis.

K$(0\ ;\ -3)$ is the y-intercept of f.
M and N are the turning points of f.



9.1	Show that the equation of f is given by $f(x) = x^3 - x^2 - 5x - 3$.	(5)
9.2	Calculate the coordinates of N.	(5)
9.3	For which values of x will:	
	9.3.1 $f(x) < 0$	(2)
	9.3.2 f be increasing	(2)
	9.3.3 f be concave up	(3)
9.4	Determine the maximum vertical distance between the graphs of f and f' in the interval $-1 < x < 0$.	(6)
		[23]

Counting Principle and Probability

(May/June 2021)

QUESTION 11

11.1 Two events, A and B, are such that:

- Events A and B are independent
- P(not A) = 0,4
- P(B) = 0,3

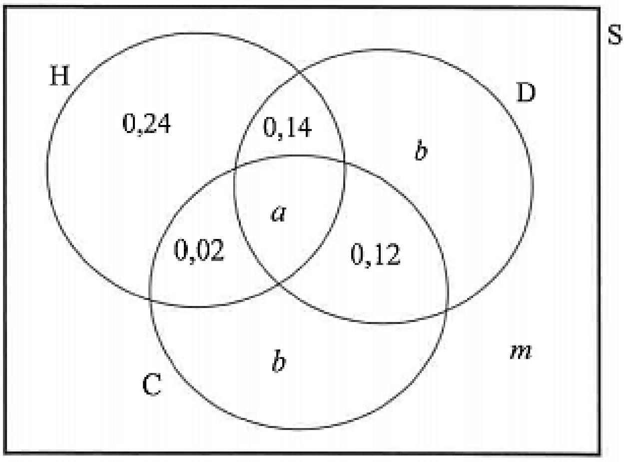
Calculate P(A and B). (3)

11.2 A survey was conducted among 150 learners at a school.

The following observations were made:

- The probability that a learner, selected at random, will take part in:
  - Only hockey (H) is 0,24
  - Hockey and debating (D), but not chess (C) is 0,14
  - Debating and chess, but not hockey is 0,12
  - Hockey and chess, but not debating is 0,02
- The probability that a learner, selected at random, participates in at least one activity is 0,7.
- 15 learners participated in all three activities.
- The number of learners that participate only in debating is the same as the number of learners who participate only in chess.

The Venn diagram below shows some of the above information.



11.2.1 Determine a, the probability that a learner, selected at random, participates in all three activities. (1)

11.2.2 Determine m, the probability that a learner, selected at random, does NOT participate in any of the three activities. (1)

11.2.3 How many learners play only chess? (4)

11.3 A three-digit number is made up by using three randomly selected digits from 0 to 9. NO digit may be repeated.

11.3.1 Determine the total number of possible three-digit numbers, greater than 100, that can be formed. (2)

11.3.2 Determine the total number of possible three-digit numbers, both even and greater than 600, that can be formed. (4)

[15]

(May/June 2018)

QUESTION 10

Ben, Nhlanhla, Owen, Derick and 6 other athletes take part in a 100 m race. Each athlete will be allocated a lane in which to run. The athletic track has 10 lanes.

10.1 In how many different ways can all the athletes be allocated a lane? (2)

10.2 Four athletes taking part in the event insist on being placed in lanes next to each other. In how many different ways can the lanes be allocated to the athletes now? (3)

10.3 If lanes are randomly allocated to athletes, determine the probability that Ben will be placed in lane 1, Nhlanhla in lane 3, Owen in lane 5 and Derick in lane 7. (2)

[7]

QUESTION 11

A survey on their preference of exercise was conducted among 140 people in two age groups. The information is summarised below.

AGE	TENNIS	RUNNING	GYM	TOTAL
35 years and younger	a	28	c	80
Older than 35 years	b	21	d	60
	21	49	70	140

11.1 If it is given that preferring to play tennis and age are independent of each other, determine the value of a. (3)

11.2 If it is given that $a = 12$, determine the probability that a randomly selected person prefers going to the gym or is in the age group 35 years and younger. (5)

[8]

(May/June 2024)

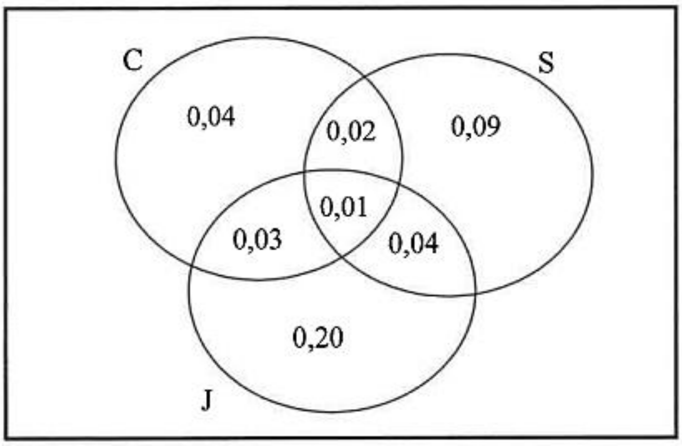
QUESTION 11

11.1 Two events, A and B, are such that:

- P(A) = 0,4
- P(A or B) = 0,52
- A and B are mutually exclusive

Calculate P(B). (2)

11.2 The items that a learner bought at a tuck shop were recorded over a period of time. The probabilities of the learner buying a sandwich (S), a chocolate (C) and a juice (J) are shown in the Venn diagram below.



11.2.1 What is the probability that the learner will buy a sandwich? (1)

11.2.2 Calculate the probability that the learner will buy at least two of the three items. (2)

11.2.3 Calculate the probability that the learner would NOT buy any of the three items. (2)

11.3 Seven guitar players, each with a different name, participate in a concert.

11.3.1 In how many different ways can the names of the guitar players be listed, one below the other, in the programme? (1)

11.3.2 After the performance, the guitar players wait backstage. There is a bench with only room for four to sit on.

What will be the probability that the four guitar players will be sitting in alphabetical order, from left to right? (3)

11.3.3 During the performance, the seven guitar players sit in a line on stage. Four guitar players are female and three are male.

In how many different ways can they be seated if the males may not sit next to each other? (3)

[14]

(May/June 2023)

QUESTION 10

10.1 A group of people participated in a trial to test a new headache pill.

- 50% of the participants received the headache pill.
- 50% of the participants received a sugar pill.
- $\frac{2}{5}$ of the group receiving the headache pill were not cured.
- $\frac{3}{10}$ of the group receiving the sugar pill were cured.

10.1.1 Represent the given information on a tree diagram. Indicate on your diagram the probability associated with each branch as well as the outcomes. (3)

10.1.2 Determine the probability that a person chosen at random from the group will NOT be cured. (2)

10.2 Three events, A, B and C, are considered:
$P(A)=\frac{2}{5}$, $P(B)=\frac{1}{4}$ and $P(A \text{ or } B)=\frac{13}{20}$.

10.2.1 Are events A and B mutually exclusive? Support your answer with the necessary calculations. (2)

10.2.2 Determine P(only C), if it is further given that
$P(A \text{ or } C)=\frac{7}{10}$, $P(A \text{ and } C)=\frac{2}{5}$ and $2P(B \text{ and } C)=P(A \text{ and } C)$. (3)

10.2.3 Determine the probability that events A, B or C do NOT take place. (2)

10.3 Seven friends (4 boys and 3 girls) want to stand in a straight line next to each other to take a photo.

10.3.1 In how many ways can the 3 girls stand next to each other in the photo? (2)

10.3.2 In the next photo, determine the probability that Selwyn (a boy) and Lindiwe (a girl) will NOT stand next to each other in the photo. (3)

[17]

(May/June 2022)

QUESTION 10

10.1 Flags from four African countries and three European countries were displayed in a row during the 2021 Olympics.

Determine:

10.1.1 The total number of possible ways in which all 7 flags from these countries could be displayed (2)

10.1.2 The probability that the flags from the African countries were displayed next to each other (3)

10.2 A and B are two independent events.

$P(A) = 0{,}4$ and $P(A \text{ or } B) = 0{,}88$

Calculate $P(B)$. (3)

10.3 There are 120 passengers on board an aeroplane. Passengers have a choice between a meat sandwich or a cheese sandwich, but more passengers will choose a meat sandwich. There are only 120 sandwiches available to choose from. The probability that the first passenger chooses a meat sandwich and the second passenger chooses a cheese sandwich is $\frac{18}{85}$. Calculate the probability that the first passenger will choose a cheese sandwich. (5)

[13]

SECTION 3: Paper 2
Content and Examination Guidelines

STATISTICS AND REGRESSION

Measures of Central Tendency for Ungrouped Data

$Mean = \frac{sum\ of\ all\ values}{total\ number\ of\ values}$ $\bar{x} = \frac{\sum x}{n}$	Where : $\bar{x}$ = mean Σx = sum of all values n = number of values

Mode

The mode is the value that appears most frequently in a set of data points.

Median

The median is the middle number in a set of data points. position of median = $\frac{1}{2}(n+1)$

N.B Data must be arranged in ascending order before calculating the median

Range as a Measure of Dispersion

$Range = Biggest\ Values - Smallest\ Value$

Measures of Central Tendency for Grouped Data

Estimated Mean = $\frac{\sum f.x_i}{\sum f}$	f → frequency x_i → the class midpoint Calculating x_i $x_i = \frac{1}{2}(lower\ class\ limit + upper\ class\ limit)$
Modal Class	The class with the highest frequency

Class With Median

- Position of median for grouped data → $\frac{1}{2}n$
- Calculating class with median → add the frequencies from the top to locate the class with the median:

Time(hours)	Frequency	
$0 \leq x < 1$	5	5 values
$1 \leq x < 2$	9	14 values
$2 \leq x < 3$	12	26 values
$3 \leq x < 4$	6	
	$n = \sum f = 32$	

The median lies here

N.B position of median $= \frac{1}{2}n = \frac{1}{2}(32) = 16$

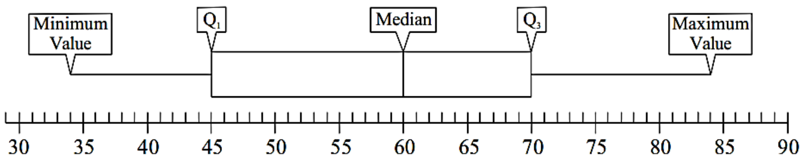
Five Number Summary and Box and Whisker Plot

FIVE NUMBER SUMMARY

1. Minimum value
2. Lower quartile Q_1
3. Median Q_2
4. Upper quartile Q_3
5. Maximum value

BOX AND WHISKER PLOT

A box and whisker plot is a visual representation of the five number summary.



Identifying Outliers

- Any data item that is

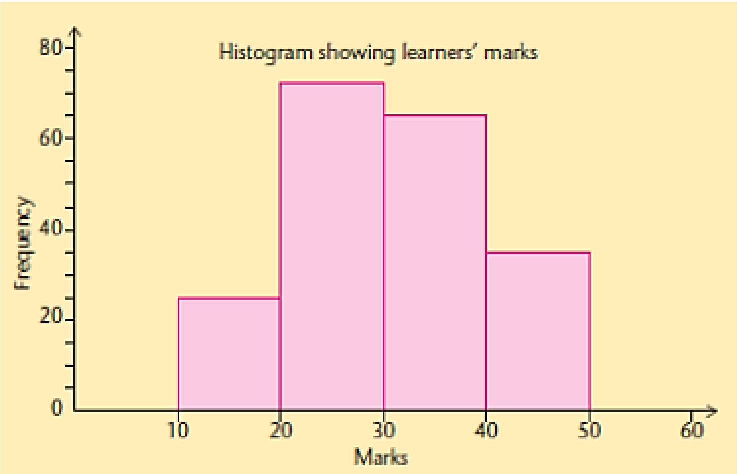
Less than $Q_1 - 1{,}5 \times IQR$
OR
More than $Q_3 + 1{,}5 \times IQR$
is an outlier.

Histograms

A histogram gives us a visual interpretation of **GROUPED DATA**. It looks very similar to a **bar graph**, but there are **NO** gabs between the bars.

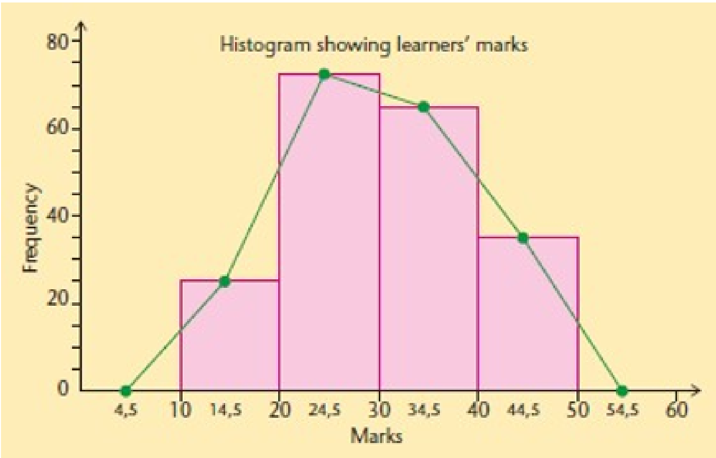
In HISTOGRAM INCLUDE THE FOLLOWING

- Tittle on top that describe what is contained in histogram
- Group/class intervals in x axis
- Frequency in y axis
- Bars with no gabs in between



Frequency Polygons

Drawn from **HISTOGRAM** by joining the **midpoints** of the top of the columns of the histogram. At the ends, extend line to the midpoints of class below lower values and the midpoint of the class above upper value to touch x axis**(grounded)**



Distribution of The Data

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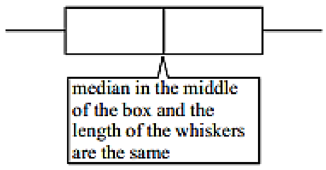
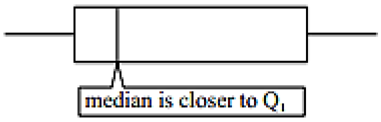
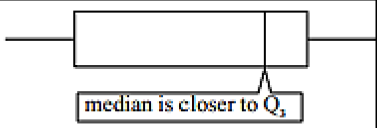
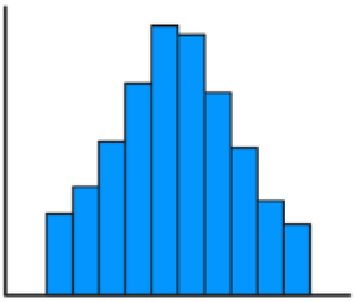
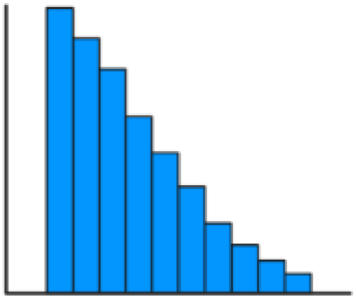
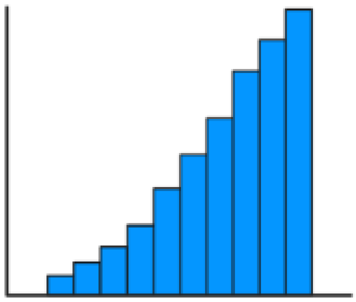
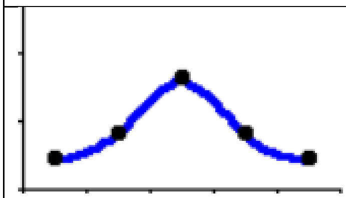
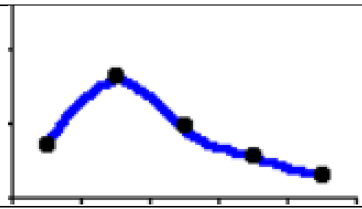
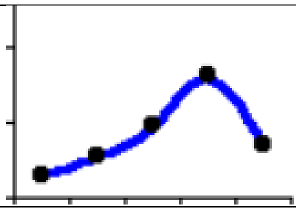
(Skewed to The Right)</th><th>Negatively Skewed
(Skewed to The Left)</th></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>Mean = Median</td><td>Mean > Median</td><td>Mean < Median</td></tr>
</table>

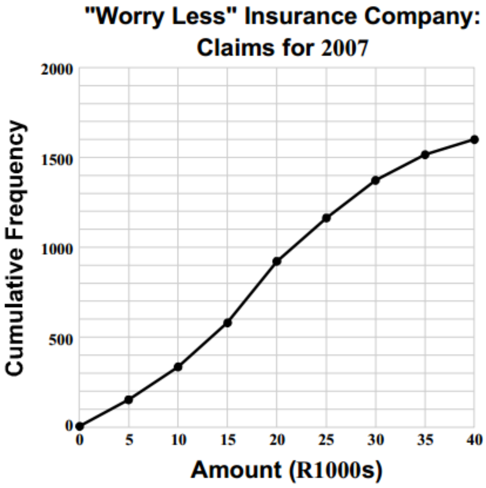
Ogives (Cumulative Frequency Curves)

To find the **cumulative frequency,**

- Add up the frequencies as you go down the frequency table.
- The last value of **cumulative frequency** must be equal to the total sum of all frequencies.

Always remember when drawing cumulative frequency curve, points to be plotted are the upper-class boundary against the cumulative frequency.

Amount Claimed (R1000s)	Upper Class Boundary	No. of Claims (frequency)	Cumulative Frequency	Points to Plot
	0	0	0	(0 ; 0)
$0 < x \leq 5$	5	150	150	(5 ; 150)
$5 < x \leq 10$	10	190	340	(10 ; 340)
$10 < x \leq 15$	15	250	590	(15 ; 590)
$15 < x \leq 20$	20	320	910	(20 ; 910)
$20 < x \leq 25$	25	260	1170	(25 ; 1170)
$25 < x \leq 30$	30	210	1380	(30 ; 1380)
$30 < x \leq 35$	35	140	1520	(35 ; 1520)
$35 < x \leq 40$	40	80	1600	(40 ; 1600)



Variance and Standard Deviation of Ungrouped Data

- Standard deviation is a measure of dispersion (spread of data) about the mean.
- Standard deviation = $\sqrt{Variance}$
    - Therefore, Variance is the $(standard\ deviation)^2$

We use the following symbol for standard deviation → σ

N.B Always use a calculator to calculate variance or standard deviation

Interpretation of Standard Deviation

- The bigger the standard deviation → The more data is spread out
- The smaller the standard deviation → The less data is spread out
- One Standard Deviation Interval → $(\bar{x} - \sigma; \bar{x} + \sigma)$
- Two Standard Deviation Interval → $(\bar{x} - 2\sigma; \bar{x} + 2\sigma)$

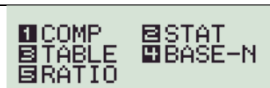
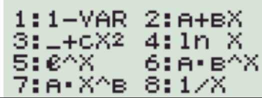
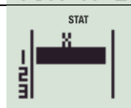
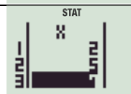
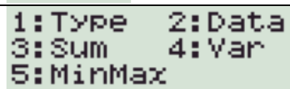
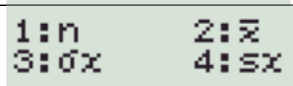
Example (Calculating mean and standard deviation (Ungrouped data))

Given data in the table below, determine the mean and standard deviation:

x	2	5	7

Solution

In the following steps we used the calculator **CASIO fx-82ZA PLUS II**

Step 1	Press **Mode** button	
Step 2	Press number **2** for **STAT**	
Step 3	Choose **1-VAR** by pressing number **1**	
Step 4	In the X-column, enter all x-values, one after the other by pressing = after each entry.	
Step 5	Press **AC** button →Press **SHIFT** button→Press the number **1**	
Step 6	Choose **Var** by pressing the number **4**	
Step 7	Press the number **2** followed by = to get the mean and press the number **3** followed by = to get the standard deviation. **N.B After getting each value, you need to repeat steps 5 – 6 to get the value in step 7. Press the AC button after getting each value.**	Mean = 4,67 Standard Deviation = 2,05

N.B to get back to the normal (Comp.) mode, press MODE button followed by number 1

Example (Calculating mean and standard deviation (Grouped data))

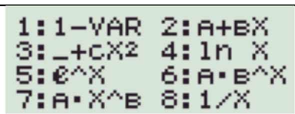
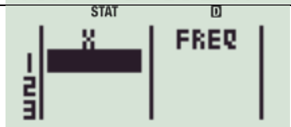
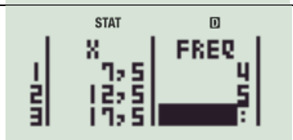
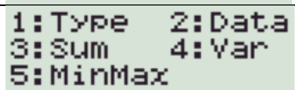
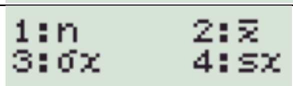
Given data in the table below, determine the estimated mean and standard deviation:

Class	Frequency
$5 < x \leq 10$	**4**
$10 < x \leq 15$	**5**
$15 < x \leq 20$	**8**

Solution

Class	Frequency	x_i (class midpoint)
$5 < x \leq 10$	**4**	**7,5**
$10 < x \leq 15$	**5**	**12,5**
$15 < x \leq 20$	**8**	**17,5**

In the following steps we used the calculator **CASIO fx-82ZA PLUS II**

Step 1	Since we are dealing with grouped data, we must first turn on the frequency feature as follows: Press **SHIFT →MODE→Down Arrow** (from **REPLAY** button)→ Choose STAT by pressing **3→ B** Choose **ON** by pressing **1.**	
Step 2	Press MODE → Choose **STAT** by pressing **2**	
Step 3	Choose **1-VAR** by pressing number **1**	
Step 4	In the X-column, enter all class midpoint values, one after the other by pressing = after each entry. Then move to the Frequency column and enter corresponding frequencies.	
Step 5	Press **AC** button →Press **SHIFT** button→Press the number **1**	
Step 6	Choose **Var** by pressing the number **4**	
Step 7	Press the number **2** followed by = to get the estimate mean and press the number **3** followed by = to get the estimate standard deviation. **N.B After getting each value, you need to repeat steps 5 – 6 to get the value in step 7. Press the AC button after getting each value.**	Estimate Mean = 13,68 Estimate Standard Deviation = 4,03

N.B to turn off the frequency follow Step 1 (but choose 2 to turn OFF)

N.B to get back to the normal (Comp.) mode, press MODE button followed by number 1

Regression Line (Line of Best Fit or The Least Squares Line)

Formula to find the predicted y −value ($\hat{y}$): $\boldsymbol{\hat{y} = a + bx}$

N.B Use a calculator to find the value of $\boldsymbol{a}$ and $\boldsymbol{b}$.

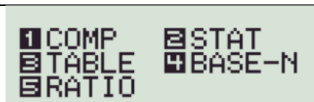
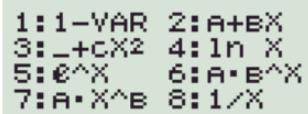
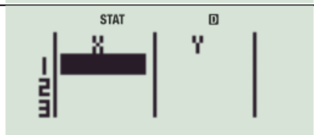
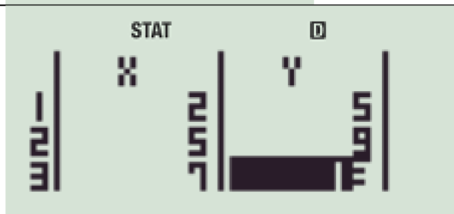
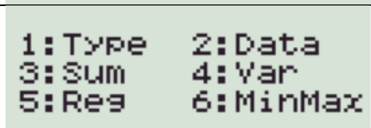
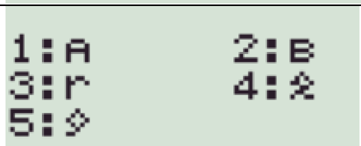
Example (Finding the equation of regression line)

Given data in the table below, determine the equation of line of best fit:

$\boldsymbol{x}$	2	5	7
$\boldsymbol{y}$	5	9	13

Solution

In the following steps we used the calculator **CASIO fx-82ZA PLUS II**

Step 1	Press **Mode** button	
Step 2	Press number **2** for **STAT**	
Step 3	Choose option **2: A+BX** by pressing the number **2**	
Step 4	In the X-column, enter all x-values, one after the other by pressing = after each entry. Then move to the Y-column, enter all y-values, one after the other by pressing = after each entry.	
Step 5	Press **AC** button →Press **SHIFT** button→Press the number **1**	
Step 6	Choose **5:Reg** by pressing the number **5**	
Step 7	Press the number **1** followed by = to get the value of **A**, and press the number **2** followed by = to get the value of **B**. You can also get the value of **r (correlation coefficient). N.B After getting each value, you need to repeat steps 5 – 6 to get the value in step 7. Press the AC button after getting each value.**	**A** = 1,63 **B** = 1,58 **r** = 0,99

The equation of line of best fit is then $\boldsymbol{\hat{y} = 1,63 + 1,58x}$

N.B to get back to the normal (Comp.) mode, press MODE button followed by number 1

Drawing the line of best fit

Approach 1

Substitute any two x −values from the table to get the predicted y −values. Then plot the two points and join them.

N.B Ensure that your line is long enough to cover the

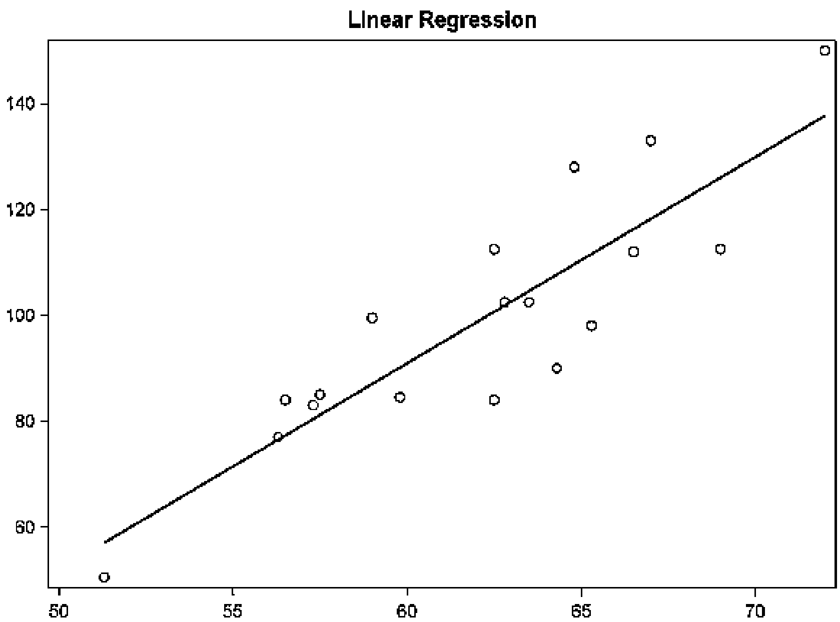
Approach 2

Determine the mean point and the y-intercept, then join the two points.

To determine the mean point, use the example on the previous page, follow steps 1 – 5, then the following steps:

Step 6	Choose **4:Var** by pressing the number **4**	1:n 2:$\bar{x}$ 3:σx 4:sx 5:$\bar{y}$ 6:σy 7:sy
Step 7	Press the number **2** followed by = to get the mean for $\boldsymbol{x}$, and press the number **5** followed by = to get the mean for $\boldsymbol{y}$. **N.B After getting each value, you need to repeat steps 5 – 6 to get the value in step 7. Press the AC button after getting each value.**	$\bar{x} = 4,67$ $\bar{y} = 9$ $\therefore$ ***mean point is*** $(4,67\ ;\ 9)$

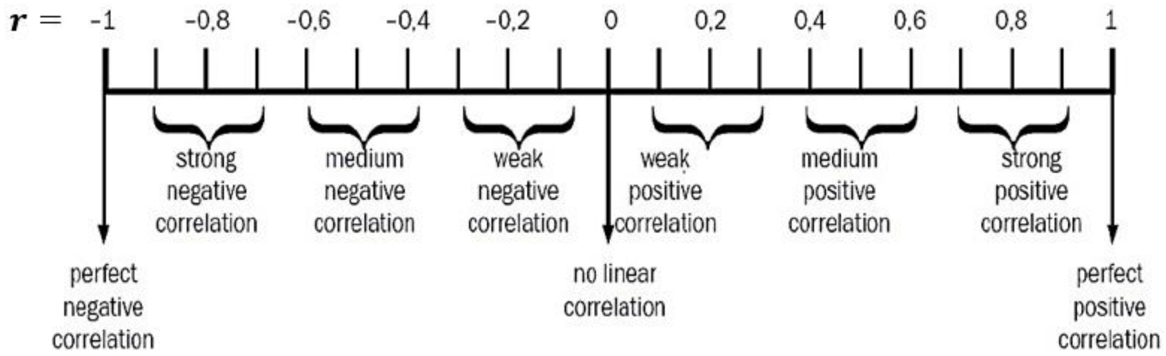
N.B When drawing the line of best fit, ensure that the line is long enough to cover the grid, see below:



Correlation Coefficient

- Correlation coefficient (r) indicates the strength of the relationship between the two variables. **Use calculator to calculate *r*** ***(See example on page 80)*** **.**
- $-1 \leq r \leq 1$

Reading/Describing Correlation Coefficient



Examination Guidelines (Statistics)

Source: Mathematics Examination Guidelines Grade 12, 2021

1. Candidates should be encouraged to use the calculator to calculate standard deviation, variance and the equation of the least squares regression line.
2. The interpretation of standard deviation in terms of normal distribution is not examinable.
3. Candidates are expected to identify outliers intuitively in both the scatter plot as well as the box and whisker diagram.

   In the case of the box and whisker diagram, observations that lie outside the interval (lower quartile – 1,5 IQR; upper quartile + 1,5 IQR) are considered to be outliers. However, candidates will not be penalised if they did not make use of this formula in identifying outliers.

ANALYTICAL GEOMETRY

Straight Lines

① Distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

② Midpoint formula:

$$M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

③ The gradient / slope of a line:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad \frac{y_1 - y_2}{x_1 - x_2}$$

order is very important! You HAVE to use the same order in the numerator and denominator.

This line is increasing. It has a positive gradient.

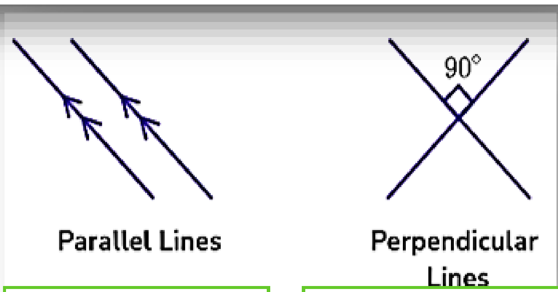
This line is decreasing. It has a negative gradient.

④ The equation of a straight line:

A straight line equation is written in the form $y = mx + c$.

$$y = \underbrace{m}_{\text{gradient}}x + \underbrace{c}_{\text{y-intercept}} \quad \text{or} \quad y - y_1 = \underbrace{m}_{\text{gradient}}(x - x_1)$$

$(x_1 ; y_1)$: a point on the line



Parallel lines

-> same gradient

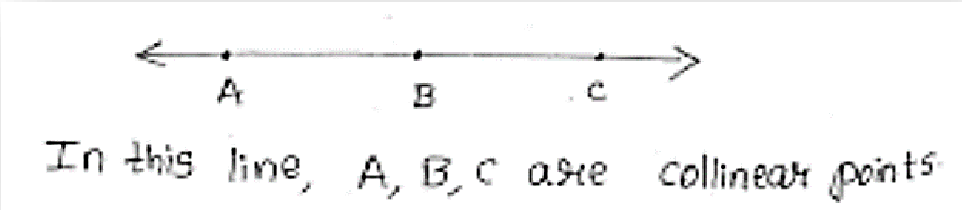
$\therefore m_1 = m_2$

Perpendicular lines

-> Product of gradients = -1

$\therefore m_1 \times m_2 = -1$

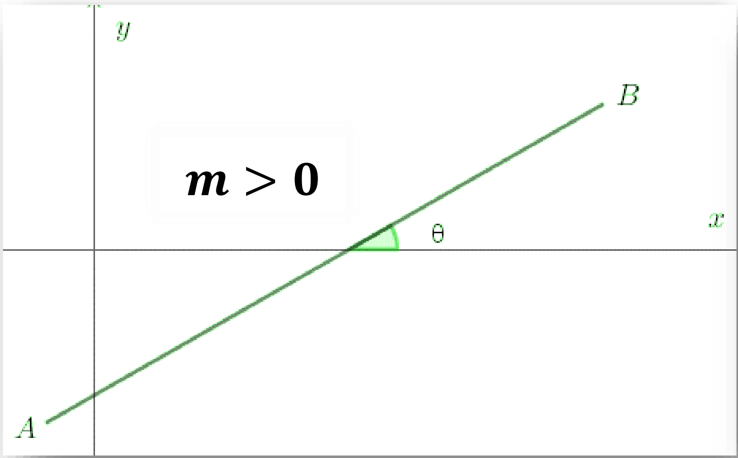
Collinear points → Two or more points are called collinear if all points lie on the same straight line.

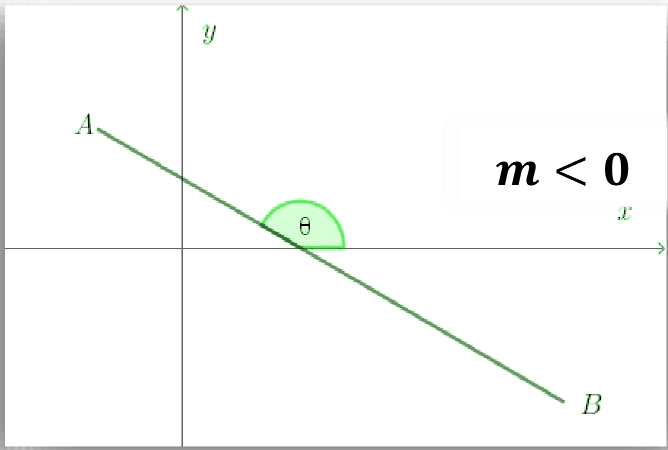


$$\therefore m_{AB} = m_{BC} = m_{AC}$$

Inclination of a line

$$m = \tan\theta$$

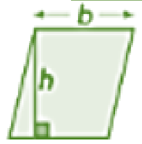
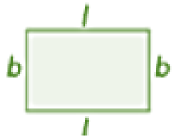
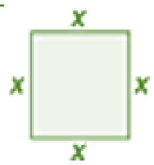
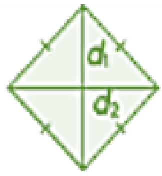
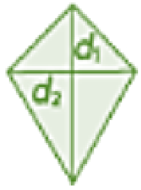
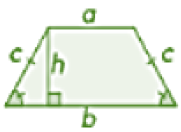




$$\theta = \tan^{-1}(m)$$

$$\theta = \tan^{-1}(m) + 180°$$

Properties of Quadrilaterals

Quadrilateral	Shape	Properties	Area
Parallelogram		• Opposite sides parallel • Opposite sides equal • Opposite angles equal • Diagonals bisect each other	$b \times h$
Rectangle		• All properties of parallelogram • All angles are right angles • Diagonals are equal in length	$l \times b$
Square		• All properties of rectangle • All sides are equal • Diagonals bisect at right angle • Diagonals bisect corner angles	x^2
Rhombus		• All properties of a parallelogram • All sides are equal • Diagonals bisect at right angle • Diagonals bisect corner angles	$\frac{1}{2} \times d_1 \times d_2$
Kite		• Two pairs of adjacent sides are equal • One pair of opposite angles are equal • One diagonal bisect the other at right angle • One diagonal bisects corner angles	$\frac{1}{2} \times d_1 \times d_2$
Trapezium		One pair of opposite sides parallel	$\frac{1}{2} \times (a + b) \times h$

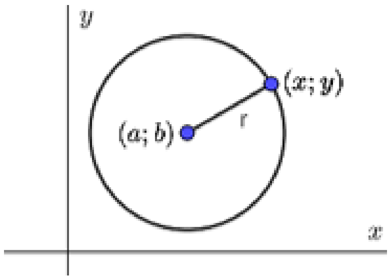
Equation of A Circle

Standard Form:

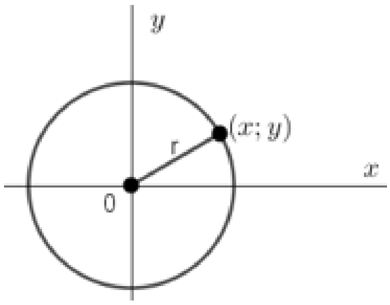
$$(x-a)^2+(y-b)^2=r^2$$

$$centre=(a;b)$$

$$radius=\sqrt{r^2}=r$$



centre at the origin $(0;0)$ **:** $x^2+y^2=r^2$



General form:

$$x^2+y^2+cx+dy+e=0$$

Given the equation of the circle $x^2+y^2-6x+8y+7=0$

Determine the centre and the radius:

$$x^2-6x+y^2+8y=-7$$

$$x^2-6x+(\square)^2+y^2+8y+(\square)^2=-7+(\square)^2+(\square)^2$$

$$x^2-6x+(-3)^2+y^2+8y+(4)^2=-7+(-3)^2+(4)^2$$

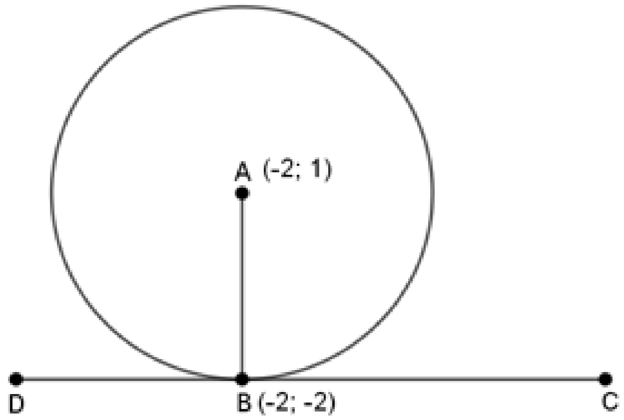
$$(x-3)^2+(y+4)^2=18$$

$$centre=(3;-4)$$

$$radius=3\sqrt{2}$$

Determining the equation of a tangent to the circle

A is the Centre of the circle. DC is a tangent to the circle at B.



1. Determine the gradient of the radius AB

2. AB ⊥DC (tangent ⊥ radius)

3. Determine the gradient of tangent DC

$$m_1\times m_2=-1$$

4. Then find the equation of the tangent DC

$$y=mx+c$$

Examination Guidelines (Statistics)

Source: Mathematics Examination Guidelines Grade 12, 2021

1. Prove the properties of polygons by using analytical methods.
2. The concept of collinearity must be understood.
3. Candidates are expected to be able to integrate Euclidean Geometry axioms and theorems into Analytical Geometry problems.
4. The length of a tangent from a point outside the circle should be calculated.
5. Concepts involved with concurrency will not be examined.

TRIGONOMETRY

FORMULAE FOR TRIGONOMETRY

Provided in the Information Sheet

$$In\ \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha-\beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

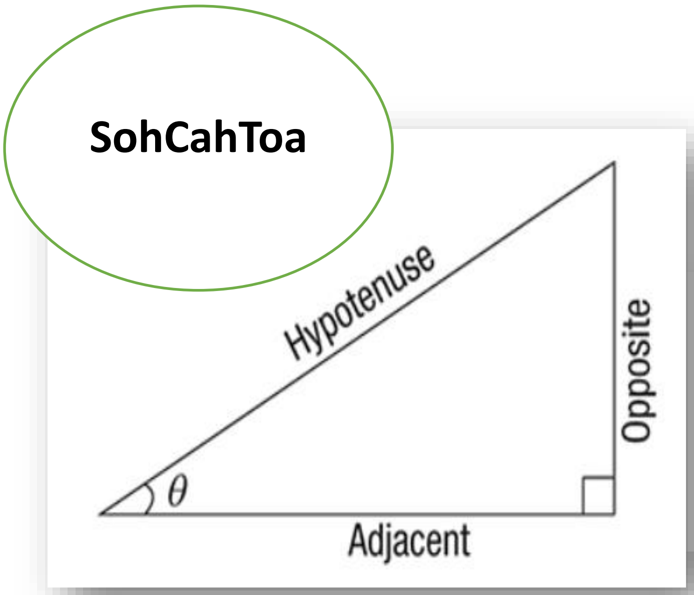
$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

Not provided in the Information Sheet

$$\boldsymbol{sin^2\alpha + cos^2\alpha = 1} \qquad \mathbf{tan}\,\alpha = \frac{\boldsymbol{sin\alpha}}{\boldsymbol{cos\alpha}}$$

$$\boldsymbol{sin^2\alpha = 1 - cos^2\alpha}$$
$$\boldsymbol{cos^2\alpha = 1 - sin^2\alpha}$$

Right-Angled Triangle

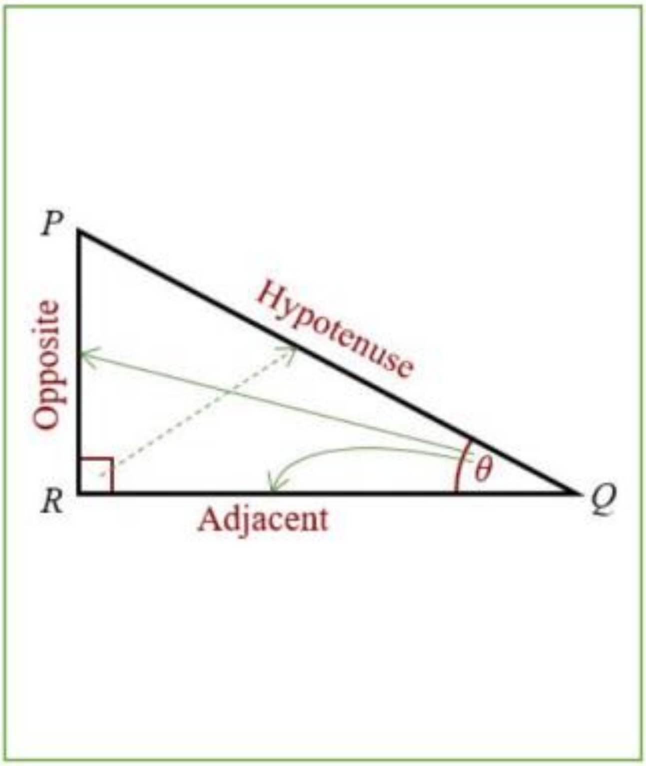


$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Theorem of Pythagoras



N.B: The Pythagoras Theorem

$$PQ^2 = PR^2 + QR^2$$

(THE HYPOTENUSE SQUARED IS EQUAL TO THE SUM OF THE SQUARES OF THE OTHER TWO SIDES)

N.B: Pythagoras Theorem (in terms of $x, y\ and\ r$)

$$x^2 + y^2 = r^2$$

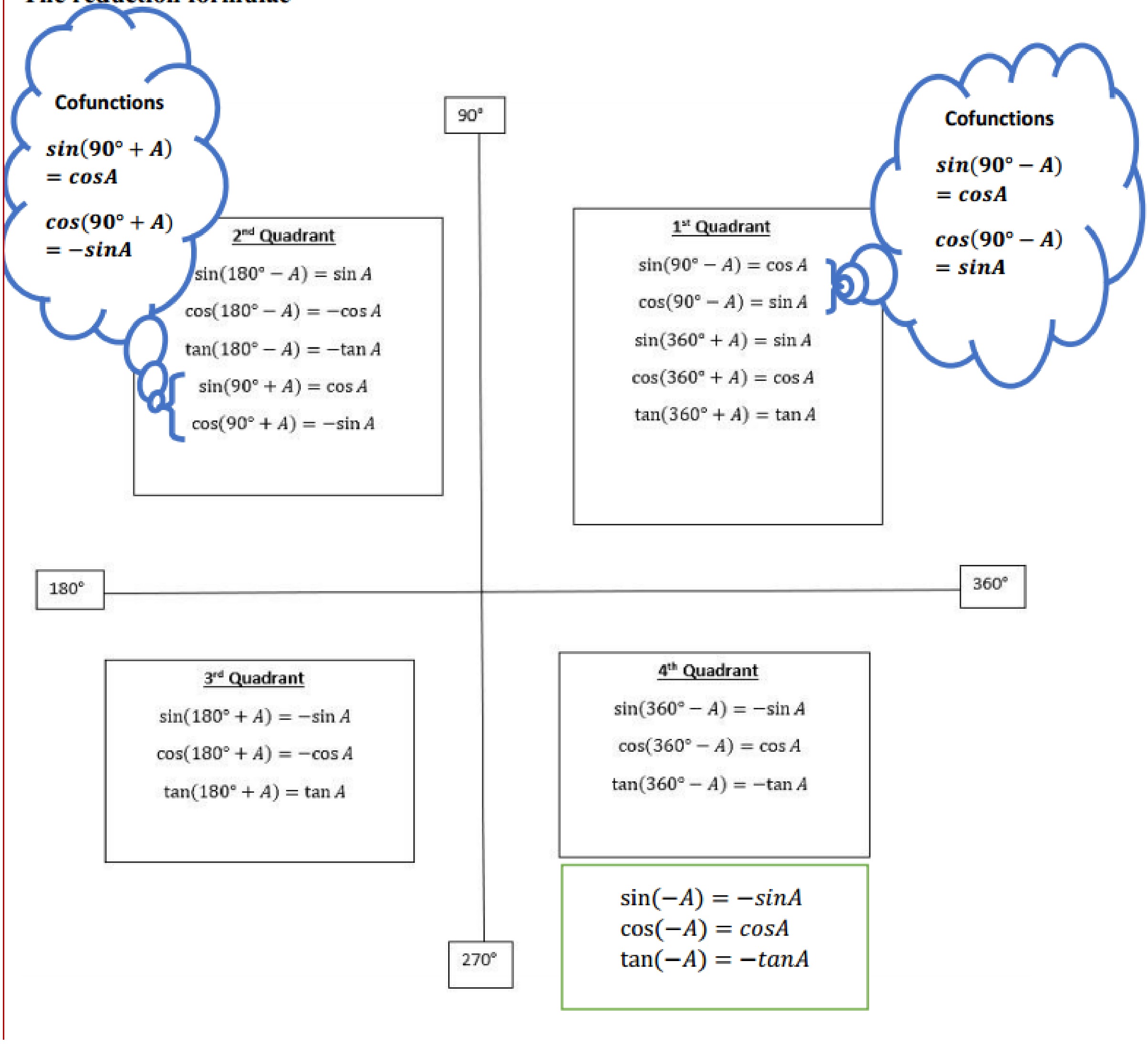
Reduction Formulae

Quadrant 1	Quadrant 2	Quadrant 3	Quadrant 4
Sine +	Sine +	Sine –	Sine –
Cosine +	Cosine –	Cosine –	Cosine +
Tangent +	Tangent –	Tangent +	Tangent –

All | **Students (sin)** | **Take (tan)** | **Coffee (cos)**

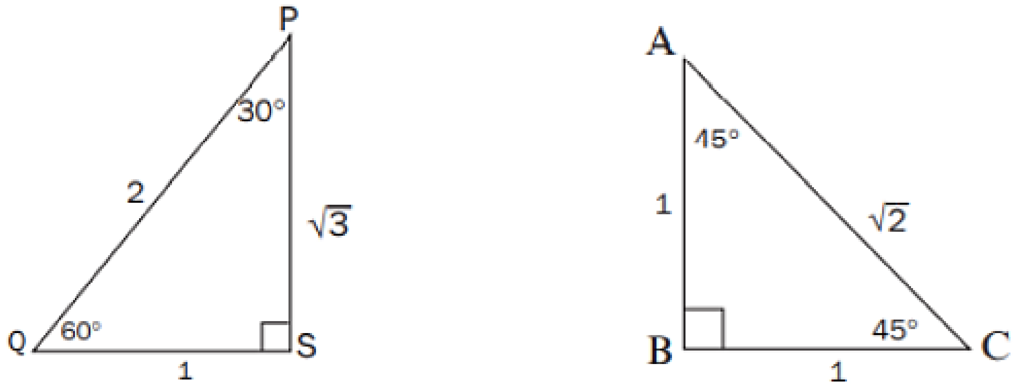
Mnemonic to help you remember

The reduction formulae



Special Angles

The following are ratios of the special angles (30°, 45° and 60°)



If you find it difficult to remember the diagrams, then learn this summary of the special angles.

θ	30°	45°	60°
sin θ	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan θ	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Identities

Grade 11 revision

The square identity formula	The quotient identity formula
$\underbrace{\sin^2\alpha + \cos^2\alpha = 1}$ $\sin^2\alpha = 1 - \cos^2\alpha$ $\cos^2\alpha = 1 - \sin^2\alpha$	$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

N.B: The above formulae will not be given on the information sheet; you must learn them by heart.

Grade 12 Identities (provided on the information sheet)

Compound Angle Formulas	$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$	$\sin(\alpha-\beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$ $\cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$
Double Angle Formulas	$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$	$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

N.B: if you come across questions with double or compound angles with the tangent ratio, use the quotien identity formula:

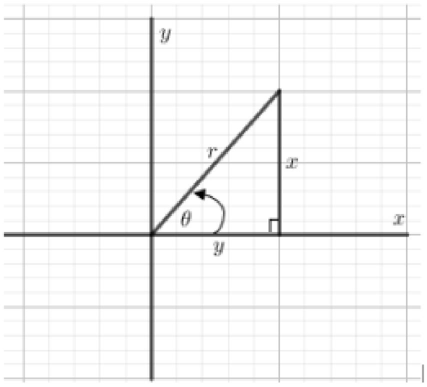
1. $\tan 2\alpha = \dfrac{\sin 2\alpha}{\cos 2\alpha}$

2. $\tan(A-B) = \dfrac{\sin(A-B)}{\cos(A-B)}$

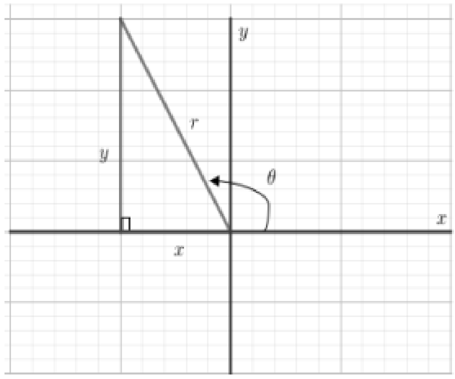
Cartesian Plane

N.B θ *is an angle measured from the positive x − axis to the termila arm (radius)*

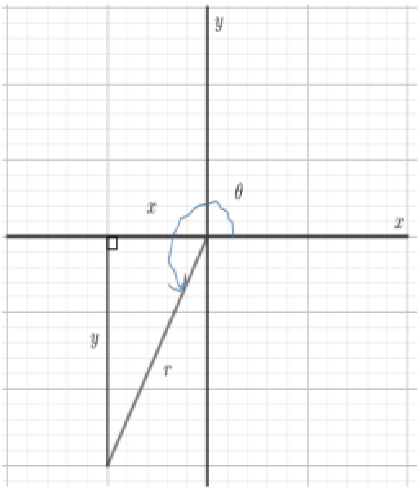
Quadrant 1



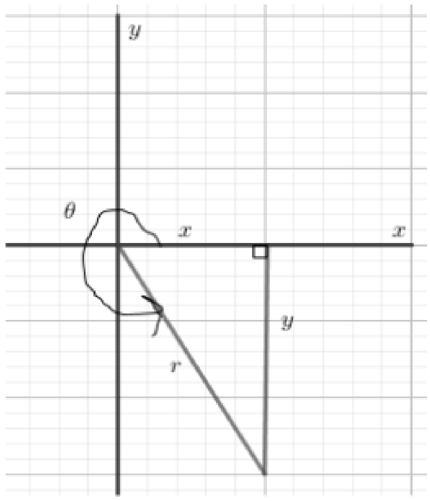
Quadrant 2



Quadrant 3



Quadrant 4



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

N.B: Pythagoras Theorem

$x^2 + y^2 = r^2$

Ratios With a Letter

Example 1

If, $sin31° = p$ determine, without using a calculator, the following in terms of p:

a. $\sin 149°$
b. $\cos(-59°)$
c. $cos62°$
d. $sin59°$

Use the reduction formula:
$$\boldsymbol{sin(180° - \alpha) = sin\alpha}$$
$$\boldsymbol{sin149° = sin(180° - 31°) = sin31°}$$

Solution

Method 1

a.
$\sin 149° = \sin 31° = p$

b.
$\cos(-59°) = \cos 59° = \sin 31° = p$

c.
$\cos 62° = 1 - 2\sin^2 31° = 1 - 2p^2$

Use the reduction formula:
$$\boldsymbol{cos(90° - \alpha) = sin\alpha}$$
$\therefore$
$$\boldsymbol{cos59° = cos(90° - 31°) = sin31°}$$

Use the double angle formula
$$\boldsymbol{cos2\alpha = 1 - 2\sin^2\alpha}$$
$\therefore$
$$\boldsymbol{cos62° = cos2(31°) = 1 - 2\sin^2 31°}$$

d.
$$\boldsymbol{sin^2 59° + cos^2 59° = 1}$$
$$\boldsymbol{sin^2 59 = 1 - cos^2 59}$$
$$\boldsymbol{sin59° = \sqrt{1 - cos^2 59}}$$
$$\boldsymbol{sin59° = \sqrt{1 - p^2}}$$

We have already shown in b. that, $\boldsymbol{cos59° = sin31° = p}$
$\boldsymbol{\therefore cos^2 59° = p^2}$

Trigonometric Equations

Use the following formulae to determine the general solutions of trigonometric equations:

If $sin\alpha = m$

Take note that, when solving trigonometric equations, after isolating the trigonometric ratio, in this case $sin\alpha = m$, then you will only have solutions when $-1 \leq m \leq 1$ (see the mother graph sine on page 39). BUT, when $m > 1$ or $m < -1$, there will be no solution (see example 7.c on page 34).

Formula	$\alpha = \sin^{-1}(m) + k.360°$ or $\alpha = 180° - \sin^{-1}(m) + k.360°$, $k \in Z$
Example 1, determine the genral solution of: $sinx = \frac{1}{2}$	$x = \sin^{-1}\left(\frac{1}{2}\right) + k.360°$ or $x = 180° - \sin^{-1}\left(\frac{1}{2}\right) + k.360°$ $x = 30° + k.360°$ or $x = 150° + k.360°$, $k \in Z$
Example 2, determine the genral solution of: $sinx = -\frac{1}{2}$	$x = \sin^{-1}\left(-\frac{1}{2}\right) + k.360°$ or $x = 180° - \sin^{-1}\left(-\frac{1}{2}\right) + k.360°$ $x = -30° + k.360°$ or $x = 210° + k.360°$, $k \in Z$

If $cos\alpha = m$

Take note that, when solving trigonometric equations, after isolating the trigonometric ratio, in this case $cos\alpha = m$, then you will only have solutions when $-1 \leq m \leq 1$ (see the mother graph of cosine on page 41). BUT, when $m > 1$ or $m < -1$, there will be no solution (see example 7.b on page 33).

Formula	$\alpha = cos^{-1}(m) + k.360°$ or $\alpha = -cos^{-1}(m) + k.360°$, $k \in Z$
Example 3, determine the genral solution of: $cosx = 1$	$x = cos^{-1}(1) + k.360°$ or $x = -\cos^{-1}(1) + k.360°$ $x = 0° + k.360°$ or $x = 0° + k.360°$ $\therefore x = 0° + k.360°$, $k \in Z$
Example 4, determine the genral solution of: $cosx = -1$	$x = cos^{-1}(-1) + k.360°$ or $x = -\cos^{-1}(-1) + k.360°$, $k \in Z$ $x = 180° + k.360°$ or $x = -180° + k.360°$, $k \in Z$

If $\tan\alpha = m$

Formula	$\alpha = tan^{-1}(m) + k.180°$, $k \in Z$
Example 5, determine the genral solution of: $tanx = 3$ **(Round your answer to TWO decimal places)**	$x = tan^{-1}(3) + k.180°$ $x = 71{,}57° + k.180°$, $k \in Z$
Example 6, determine the genral solution of: $tanx = -3$ **(Round your answer to TWO decimal places)**	$x = tan^{-1}(-3) + k.180°$ $x = -71{,}57° + k.180°$, $k \in Z$

N.B: You may use other methods to determine the general solutions, however, the above method is highly recommeded

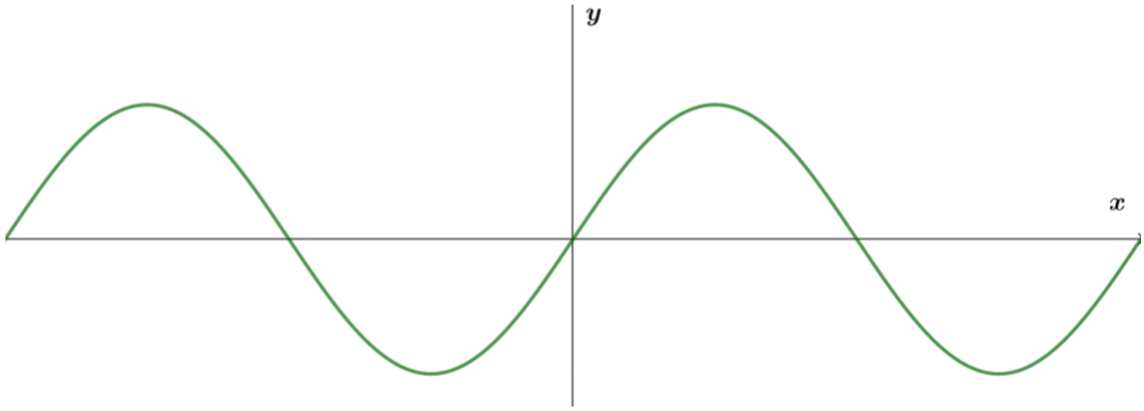
Trigonometric Functions

Summary of The sine function $y = a\sin k(x+p)+q$ **Vertical translation**

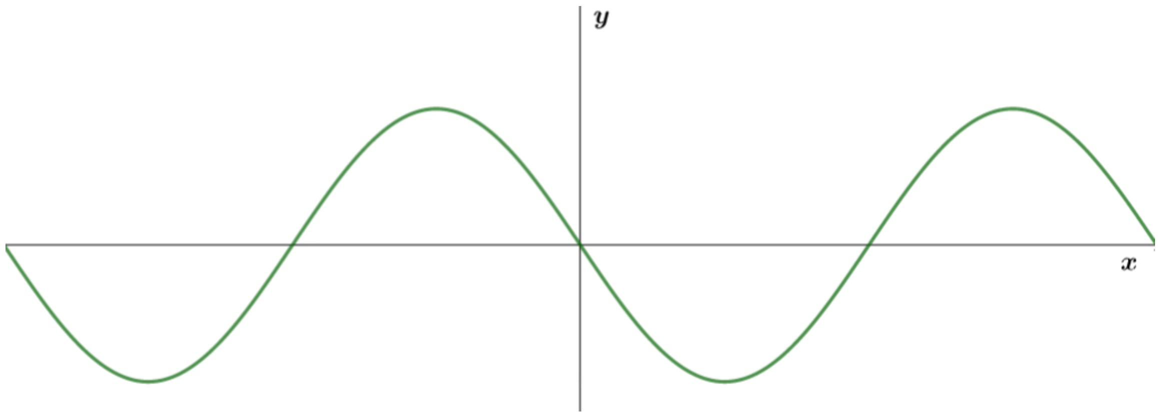
helps to find amplitude **helps with period** **horizontal translation**

- **Shape**

$\boldsymbol{a > 0}$



$\boldsymbol{a < 0}$



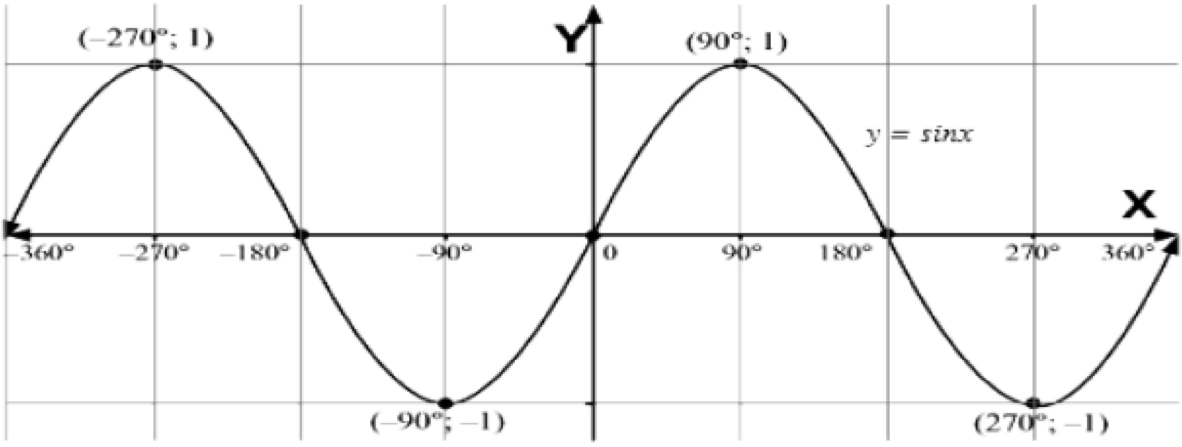
- **Amplitude** – halfway between the maximum and the minimum → $\frac{max-min}{2}$.
  - If $y = 2\sin x$, $then\ the\ amplitude\ is\ 2$
  - If $y = -3\sin x$, $then\ the\ amplitude\ is\ 3$
- $\boldsymbol{Period} = \frac{360°}{k}$
- $\boldsymbol{p} \rightarrow the\ horizontal\ shift$
  - If $y = \sin(x+45°)$ $\rightarrow shifts\ 45°\ to\ the\ left$
  - If $y = \sin(x-30°)$ $\rightarrow shifts\ 30°\ to\ the\ right$
- $\boldsymbol{q} \rightarrow the\ vertical\ shift$
  - If $y = \sin x + 3$ $\rightarrow shifts\ 3\ units\ up$
  - If $y = \sin x - 2$ $\rightarrow shifts\ 2\ units\ down$

Example 1

$sketch\ the\ graph\ y = sinx \quad for\ x \in [-360°; 360°]$

Solution

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	0	1	0	-1	0	1	0	-1	0



Take note of the following key aspects of the graph of $y = sinx \quad for\ x \in [-360°; 360°]$

1	**Maximum Value**	**1** (at $x = -270°$ and $x = 90°$)
	Minimum Value	-1 (at $x = -90°$ and $x = 270°$)
2	**Domain**	$x \in [-360°; 360°], x \in R$
	Range	$y \in [-1; 1], y \in R$
3	**x-intercepts**	$-360°, -180°, 0°, 180°, 360°$
	y-intercept	**0**
4	**Amplitude**	**1** $\left\{\frac{max-min}{2} \rightarrow \frac{1-(-1)}{2} = 1\right\}$
5	**Period**	$360°$ $\left\{period = \frac{360°}{k} \rightarrow \frac{360°}{1} = 360°\right\}$

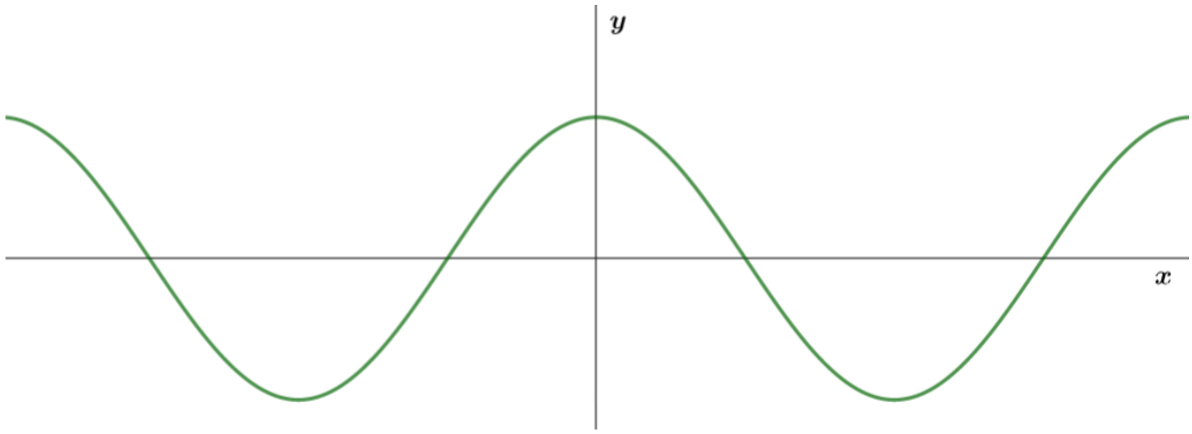
Summary of The cosine function $y = acosk(x+p)+q$

vertical translation

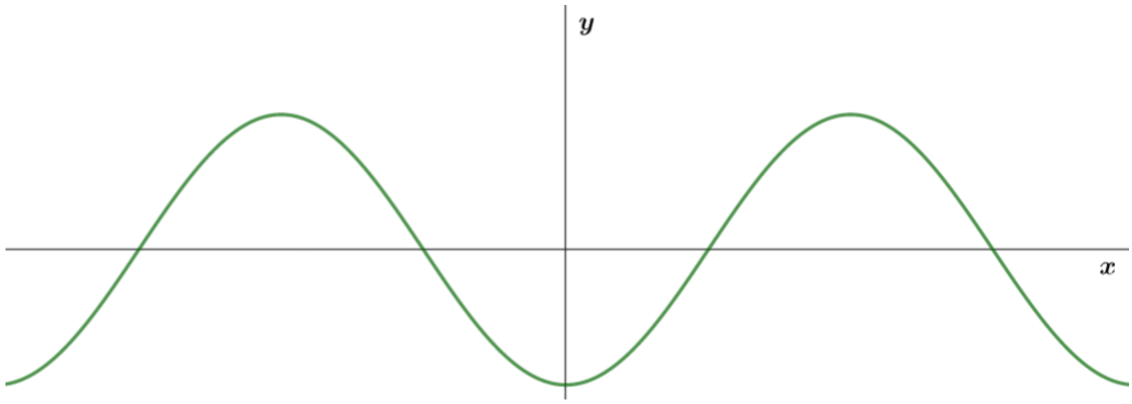
helps to find amplitude   helps with period   horizontal translation

- **Shape**

$\boldsymbol{a > 0}$



$\boldsymbol{a < 0}$



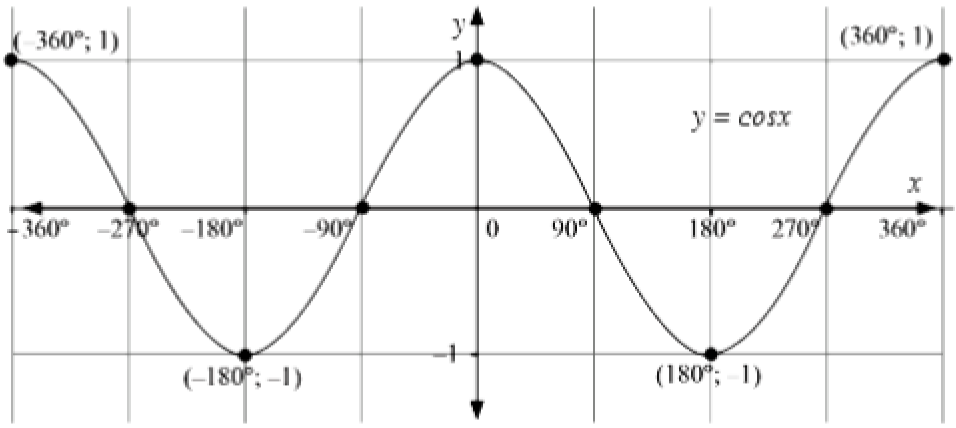
- **Amplitude** – halfway between the maximum and the minimum $\rightarrow \frac{\boldsymbol{max-min}}{\boldsymbol{2}}$.
  - If $y = 2cosx$, then the amplitude is 2
  - If $y = -3cosx$, then the amplitude is 3
- $\boldsymbol{Period} = \frac{360°}{k}$
- $\boldsymbol{p} \rightarrow$ *the horizontal shift*
  - $y = \cos(x+45°) \rightarrow shifts\ 45°\ to\ the\ left$
  - $y = \cos(x-30°) \rightarrow shifts\ 30°\ to\ the\ right$
- $\boldsymbol{q} \rightarrow$ *the vertical shift*
  - $y = cos\ x + 3 \rightarrow shifts\ 3\ units\ up$
  - $y = \cos\ x - 2 \rightarrow shifts\ 2\ units\ down$

Example 2

$sketch\ the\ graph\ y = cosx \quad for\ x \in [-360°; 360°]$

Solution

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	1	0	-1	0	1	0	-1	0	1



Take note of the following key aspects of the graph of $y = cosx \quad for\ x \in [-360°; 360°]$

1	**Maximum Value**	**1** (at $x = -360°, x = 0°\ and\ x = 90°$)
	Minimum Value	-1 (at $x = -180°\ and\ x = 180°$)
2	**Domain**	$x \in [-360°; 360°], x \in R$
	Range	$y \in [-1; 1], y \in R$
3	**x-intercepts**	$-270°, -90°, 90°, 270°$
	y-intercept	**1**
4	**Amplitude**	**1** $\{\frac{max-min}{2} \to \frac{1-(-1)}{2} = 1\}$
5	**Period**	$360°$ $\{period = \frac{360°}{k} \to \frac{360°}{1} = 360°\}$

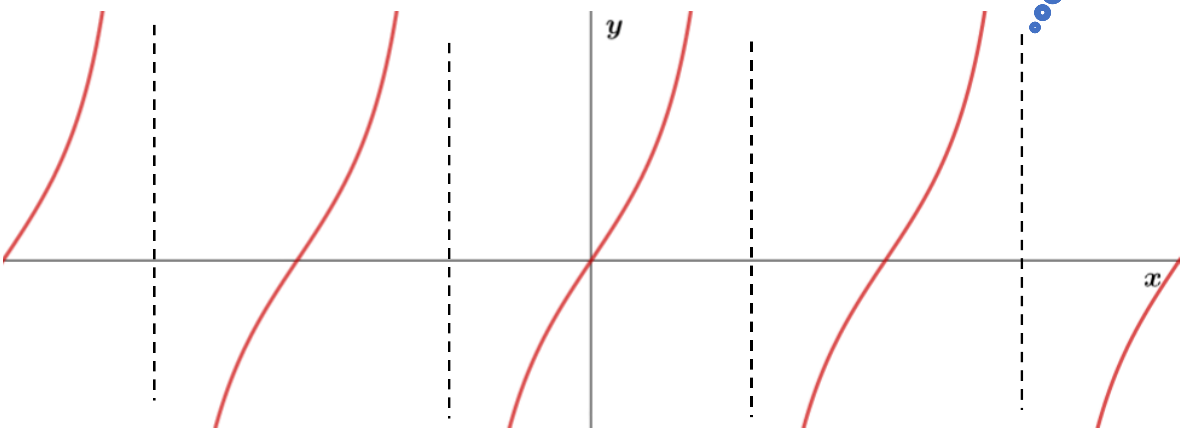
Summary of The cosine function $y = atank(x + p) + q$

helps with slope point helps with period horizontal translation vertical translation

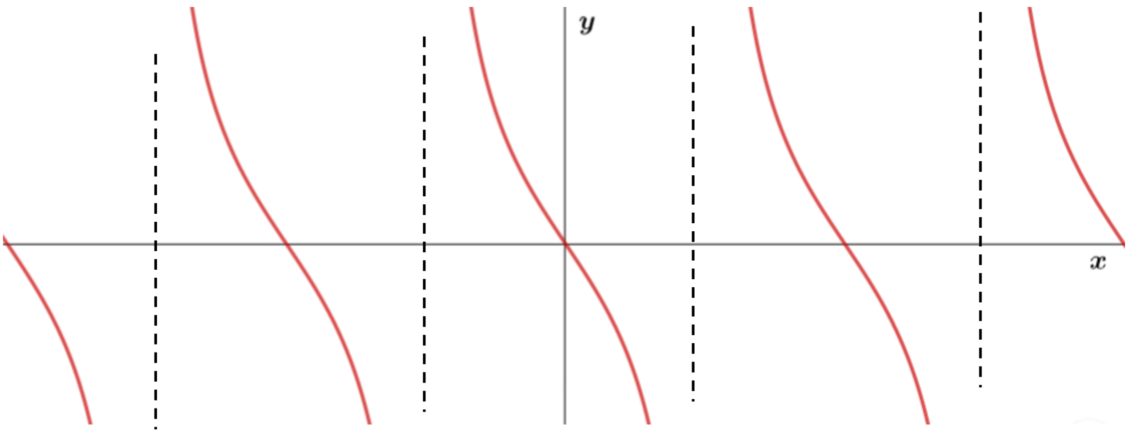
- **Shape**

Asymptote line

$a > 0$



$a < 0$



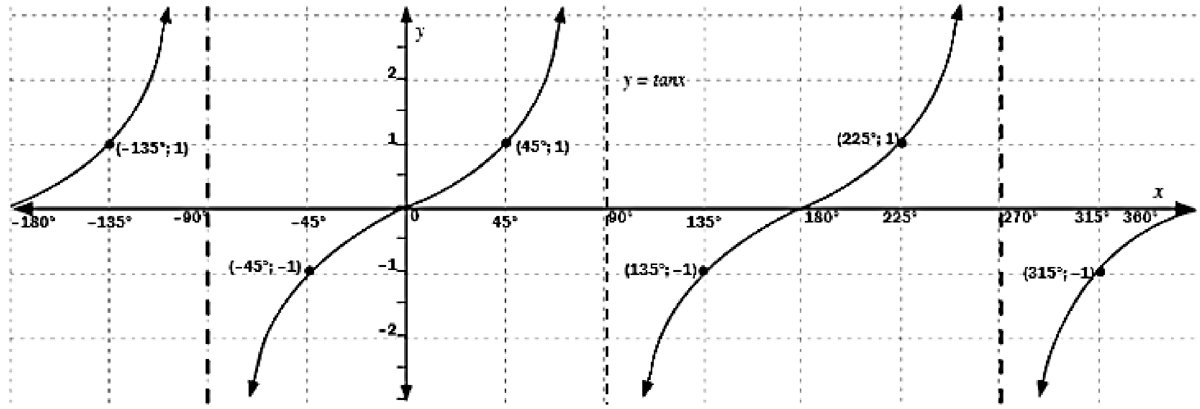
- **Amplitude** – tangent graph does not have maximum or the minimun VALUE, thus **THERE IS NO AMPLITUDE** for the tangent function.
- **Asymptotes**
  - Positions of the first asymptotes are at $0° \pm \frac{period}{2}$
  - Then, other asymptotes are found every period.
- $Period = \frac{180°}{k}$
- $p \to the\ horizontal\ shift$
  - $y = \tan\,(x + 45°)\ \to shifts\ 45°\ to\ the\ left$
  - $y = \tan\,(x - 30°)\ \to shifts\ 30°\ to\ the\ right$
- $q \to the\ vertical\ shift$
  - $y = tan\ x + 3\ \to shifts\ 3\ units\ up$
  - $y = \tan\ x - 2 \to shifts\ 2\ units\ down$

Example 3

sketch the graph $y = tanx \quad for\ x \in [-180°; 360°]$

Solution

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$
y	0	1	unde-fined	-1	0	1	unde-fined	-1	0	1	unde-fined	-1	0



Take note of the following key aspects of the graph of $y = tanx \quad for\ x \in [-180°; 360°]$

1	Maximum Value	N/A
	Minimum Value	N/A
2	Domain	$x \in [-180°; 360°], \quad but\ x \neq -90°, 90°, 270°$
	Range	$y \in (-\infty; \infty), y \in R$
3	x-intercepts	$-180°, 0°, 180°, 360°$
	y-intercept	0
4	Amplitude	N/A
5	Period	$180° \quad \{period = \frac{180°}{k} \rightarrow \frac{180°}{1} = 180°\}$
6	Equations of asymptotes	$x = -90°, x = 90°, \quad and\ x = 270$
7	Slope points	$(-135°; 1), (-45°; -1), (45°; 1), (135°; -1), (225°; 1), (315°; -1)$

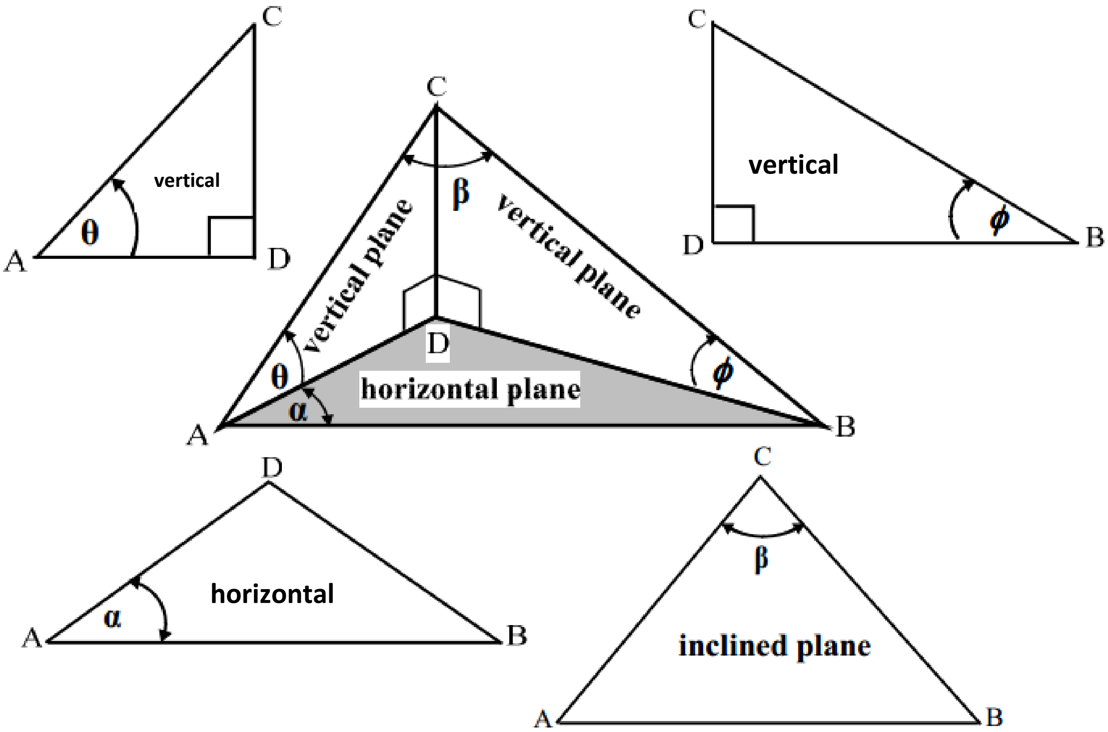
Solution of Triangles (2-D and 3-D)

3-dimensional space takes up 3 planes (horizontal, vertical and inclined/slanted).

The 3-dimensional diagram below is split such that you can work separately on each 2-D plane.

The 3-D diagram below has 4 planes:

- Vertical plane $\rightarrow \Delta ADC$
- Vertical plane $\rightarrow \Delta BDC$
- Horizontal plane $\rightarrow \Delta ADB$
- Inclined plane $\rightarrow \Delta ABC$

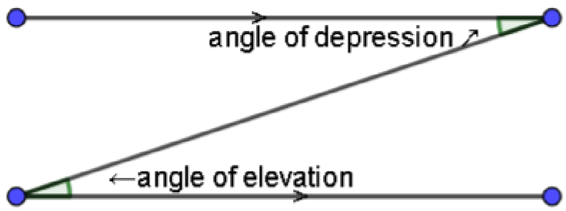


Take note that:

- You cannot add two angles from different planes to get the sum, from the 3-D diagram, $\boldsymbol{\theta + \alpha \neq C\hat{A}B}$

Angle of elevation vs Angle of depression

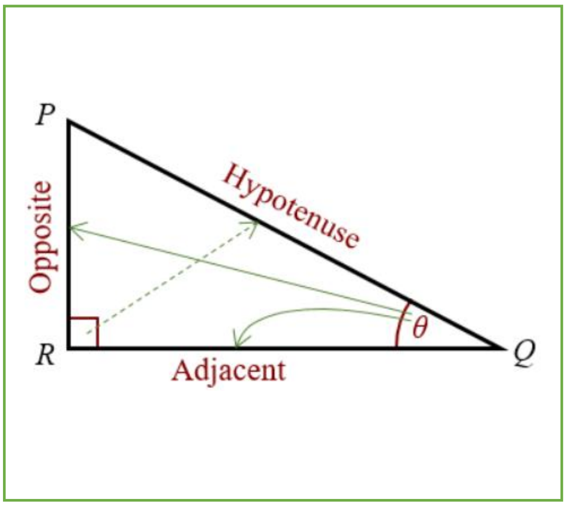
Angle of **depression** (measured from horizontal going down)



Angle of **elevation** (measured from the horizontal going up)

For right-angled triangles

Soh Cah Toa



1.

$$\text{sine of an angle } \theta = \frac{\text{length of the side opposite angle } \theta}{\text{length of the hypotenuse}}$$

$$\therefore \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

2.

$$\text{cosine of an angle } \theta = \frac{\text{length of the side adjacent to angle } \theta}{\text{length of the hypotenuse}}$$

$$\therefore \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

3

$$\text{tangent of an angle } \theta = \frac{\text{length of the side opposite angle } \theta}{\text{length of the side adjacent to angle } \theta}$$

$$\therefore \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

To calculate the area of a right-angled triangle PQR above, use the formula $Area\ \Delta = \frac{1}{2} b \times h$

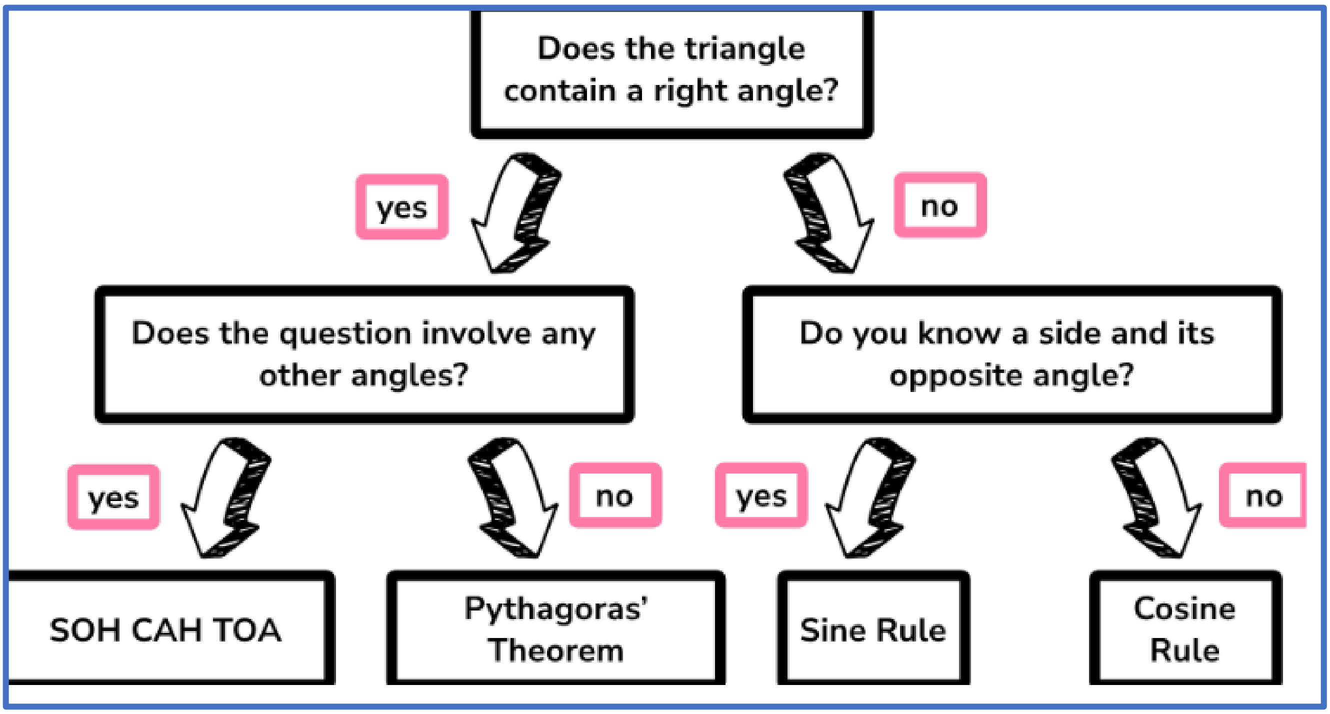
$$\therefore Area\ \Delta PQR \quad = \frac{1}{2} RQ \times PR$$

For triangles that are not right-angled triangles

Rule	Formula	When to use
Sine rule	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	Given two sides and the angle opposite one of those sides.
		One side and any two angles.
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$	Given two sides and the included angle.
		Three sides.
Area rule	$\frac{1}{2} ab \sin C$	Area is required. In order to use the formula for Area, two sides and the included angle are required.

N.B Only use area formula when you are asked to calculate the area or when you are given the area.

2-D & 3-D Approach

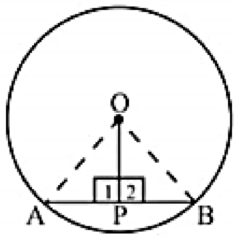


EUCLIDEAN GEOMETRY

Examinable Theorems: Know How to Prove the Following Theorems

1. **The line drawn from the centre of a circle perpendicular to a chord bisects the chord**

Given: A circle with centre O.
AB is a chord.
P is a point on AB,
such that OP $\perp$ AB.

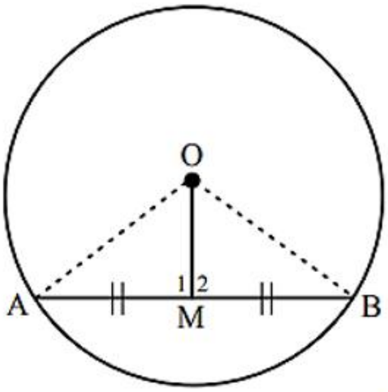


Required to prove: AP = PB.

Proof
Construction: Draw OA and OB.
In ΔOAP and ΔOBP:

OA	=	OB	(radii)
$\hat{P}_1$	=	$\hat{P}_2 = 90°$	(given)
OP	=	OP	(common side)
$\therefore \Delta OAP$	$\equiv$	ΔOBP	(RHS)
$\therefore$ AP	=	PB	($\equiv \Delta$s)

2. **The line drawn from the centre of a circle that bisects a chord is perpendicular to the chord**



Given: Circle with centre O.
Required to prove: OM $\perp$ AB

Proof
Join OA and OB
In ΔOAM and ΔOBM :

(a)	OA = OB	.....	radii
(b)	AM = BM	.....	given
(c)	OM = OM	.....	common
$\therefore \Delta OAM \equiv \Delta OBM$		.....	SSS

$\therefore \hat{M}_1 = \hat{M}_2$
But AMB is a straight line
$\therefore \hat{M}_1 = \hat{M}_2 = 90°$ ..... Adjacent supplementary $\angle s$

3. The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)

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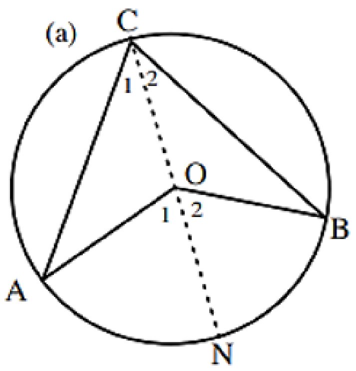
<td>
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Given: Circle with centre O :

Required to prove: $A\hat{O}B = 2A\hat{C}B$

Proof:

Join CO and produce to N.

$\hat{O}_1 = \hat{C}_1 + \hat{A}$ ..... Ext $\angle$ of ΔOAC

But $\hat{C}_1 = \hat{A}$ ..... $OA = OC$, Radii

$\therefore \hat{O}_1 = 2\hat{C}_1$

Similarly, in ΔOCB $\hat{O}_2 = 2\hat{C}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{C}_1 + 2\hat{C}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2\left(\hat{C}_1 + \hat{C}_2\right)$

$\therefore A\hat{O}B = 2A\hat{C}B$

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4. The opposite angles of a cyclic quadrilateral are supplementary

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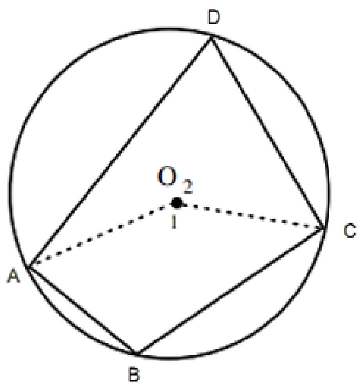
<td>
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Required to prove: $\hat{A} + \hat{C} = 180°$ and $\hat{B} + \hat{D} = 180°$

Proof

Join AO and OC.

$\hat{O}_1 = 2\hat{D}$ ..... $\angle$ at centre $= 2\times\angle$ at circum

$\hat{O}_2 = 2\hat{B}$ ..... $\angle$ at centre $= 2\times\angle$ at circum

$\hat{O}_1 + \hat{O}_2 = 2\hat{D} + 2\hat{B}$

And $\hat{O}_1 + \hat{O}_2 = 360°$ ..... $\angle$'s at a point

$\therefore 360° = 2\left(\hat{D} + \hat{B}\right)$

$\therefore 180° = \hat{D} + \hat{B}$

Similarly, by joining BO and DO, it can be proven that $\hat{A} + \hat{C} = 180°$

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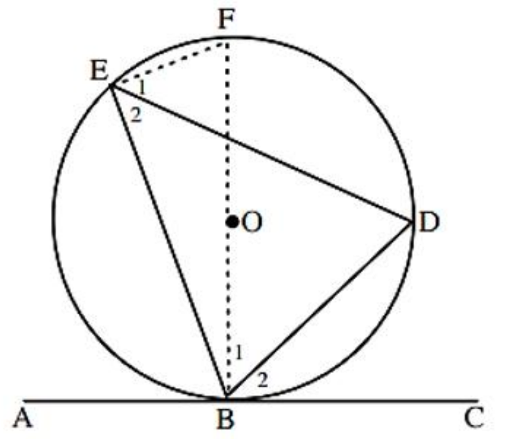
5. The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment



Given: Tangent ABC

Required to prove: $C\hat{B}D = B\hat{E}D$

<u>Proof:</u>

Draw diameter BOF and join EF

$\hat{B}_1 + \hat{B}_2 = 90°$ ….. tan ⊥ rad

$\hat{E}_1 + \hat{E}_2 = 90°$ ….. ∠ in semi-circle

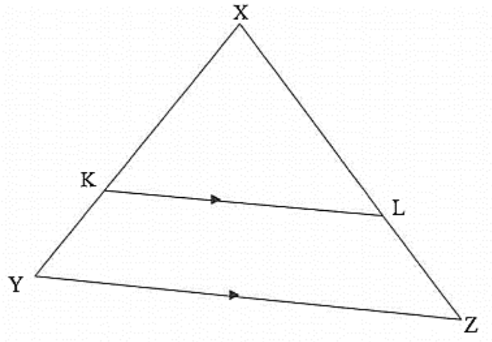
But $\hat{B}_1 = \hat{E}_1$ ….. FD subt $= \angle s$

$\therefore \hat{B}_2 = \hat{E}_2$

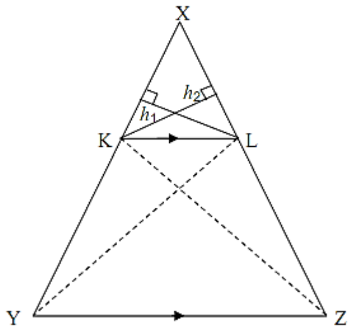
$\therefore C\hat{B}D = B\hat{E}D$

6. A line drawn parallel to one side of a triangle divides the other two sides proportionally

If $KL||YZ$, prove that $\frac{XK}{KY} = \frac{XL}{LZ}$



Proof:



Constr: Join KZ and LY and draw h_1 from $K \perp XL$ and h_2 from $L \perp XK$

Proof :

$$\frac{\text{area } \Delta XKL}{\text{area } \Delta LYK} = \frac{\frac{1}{2} XK \times h_1}{\frac{1}{2} KY \times h_1} = \frac{XK}{KY}$$

$$\frac{\text{area } \Delta XKL}{\text{area } \Delta KLZ} = \frac{\frac{1}{2} XL \times h_2}{\frac{1}{2} LZ \times h_2} = \frac{XL}{LZ}$$

area ΔXKL = area ΔXKL [common]

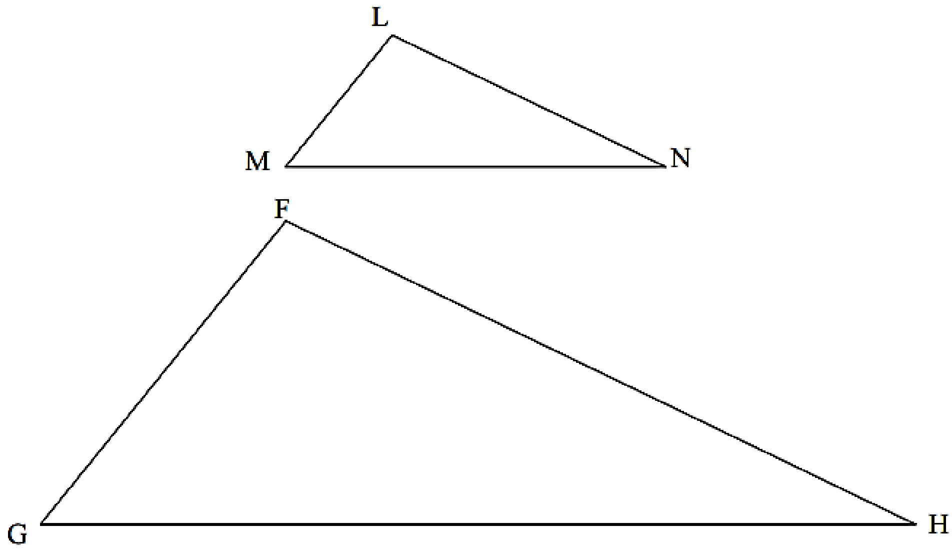
But area ΔLYK = area ΔKLZ [same base & height ; LK || YZ]

$$\therefore \frac{\text{area } \Delta XKL}{\text{area } \Delta LYK} = \frac{\text{area } \Delta XKL}{\text{area } \Delta KLZ}$$

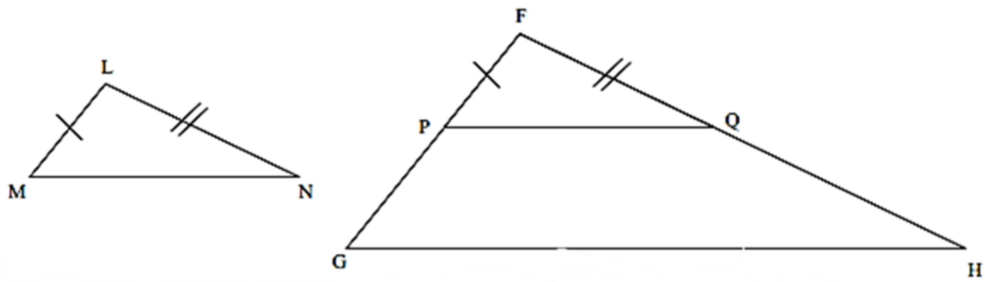
$$\therefore \frac{XK}{KY} = \frac{XL}{LZ}$$

7. Equiangular triangles are similar.

If in Δ LMN and Δ FGH it is given that $\hat{L} = \hat{F}$ and $\hat{M} = \hat{G}$, prove the theorem that states $\frac{LM}{FG} = \frac{LN}{FH}$.



Proof:



Draw a point P on FG such that FP = LM and a point Q on FH such that FQ = LN.

In ΔFPQ and ΔLMN

1. $\hat{F} = \hat{L}$ (given)
2. $FP = LM$ (construction)
3. $FQ = LN$ (construction)

$\therefore \Delta FPQ \equiv \Delta LMN$ (SAS)

$F\hat{P}Q = L\hat{M}N$ ($\equiv\Delta$s)

But $F\hat{G}H = L\hat{M}N$ (given)

$F\hat{P}Q = F\hat{G}H$

$PQ \parallel GH$ (corresponding angles =)

$\frac{FP}{FG} = \frac{FQ}{FH}$ (PQ || GH ; Prop Th)

$\frac{LM}{FG} = \frac{LN}{FH}$

Acceptable Reasons: Euclidean Geometry

Source: Grade 12 Mathematics Examination Guidelines, 2021

THEOREM STATEMENT	ACCEPTABLE REASON(S)
LINES	
The adjacent angles on a straight line are supplementary.	$\angle$s on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj $\angle$s supp
The adjacent angles in a revolution add up to 360°.	$\angle$s round a pt **OR** $\angle$s in a rev
Vertically opposite angles are equal.	vert opp $\angle$s =
If AB || CD, then the alternate angles are equal.	alt $\angle$s; AB || CD
If AB || CD, then the corresponding angles are equal.	corresp $\angle$s; AB || CD
If AB || CD, then the co-interior angles are supplementary.	co-int $\angle$s; AB || CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt $\angle$s =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp $\angle$s =
If the cointerior angles between two lines are supplementary, then the lines are parallel.	coint $\angle$s supp
TRIANGLES	
The interior angles of a triangle are supplementary.	$\angle$ sum in Δ **OR** sum of $\angle$s in Δ **OR** Int $\angle$s Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext $\angle$ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	$\angle$s opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal $\angle$s
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	Converse Pythagoras **OR** Converse Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS **OR** S$\angle$S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS **OR** $\angle\angle$S
If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent	RHS **OR** 90°HS

THEOREM STATEMENT	ACCEPTABLE REASON(S)
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side	Midpt Theorem
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt || to 2nd side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line || one side of Δ **OR** prop theorem; name || lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| Δs **OR** equiangular Δs
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	Sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height

THEOREM STATEMENT	ACCEPTABLE REASON(S)
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal ∠s
The opposite angles of a cyclic quadrilateral are supplementary	opp ∠s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp ∠s quad supp **OR** converse opp ∠s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext ∠ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext ∠ = int opp ∠ **OR** converse ext ∠ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common pt **OR** Tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem **OR** ∠ between line and chord

QUADRILATERALS	
The interior angles of a quadrilateral add up to 360°.	sum of $\angle$s in quad
The opposite sides of a parallelogram are parallel.	opp sides of ‖m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are ‖
The opposite sides of a parallelogram are equal in length.	opp sides of ‖m
If the opposite sides of a quadrilateral are equal , then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp $\angle$s of ‖m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp $\angle$s of quad are = **OR** converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ‖m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** converse diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ‖
The diagonals of a parallelogram bisect its area.	diag bisect area of ‖m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite

Examination Guidelines

Source: Grade 12 Mathematics Examination Guidelines, 2021

The following proofs of theorems are examinable:

- The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
- The line drawn from the centre of a circle that bisects a chord is perpendicular to the chord;
- The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
- The opposite angles of a cyclic quadrilateral are supplementary;
- The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;
- A line drawn parallel to one side of a triangle divides the other two sides proportionally;
- Equiangular triangles are similar.

Corollaries derived from the theorems and axioms are necessary in solving riders:

- Angles in a semi-circle
- Equal chords subtend equal angles at the circumference
- Equal chords subtend equal angles at the centre
- In equal circles, equal chords subtend equal angles at the circumference
- In equal circles, equal chords subtend equal angles at the centre.
- The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.
- If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.
- Tangents drawn from a common point outside the circle are equal in length.

The theory of quadrilaterals will be integrated into questions in the examination.

Concurrency theory is excluded.

SECTION 4: Paper 2 – Activities

PAPER 2 ACTIVITIES

Statistics and Regression

(May/June 2021)

QUESTION 1

1.1 Sam recorded the amount of data (in MB) that she had used on each of the first 15 days in April. The information is shown in the table below.

26	13	3	18	12	34	24	58	16	10	15	69	20	17	40

1.1.1 Calculate the:

(a) Mean for the data set (2)

(b) Standard deviation for the data set (1)

1.1.2 Determine the number of days on which the amount of data used was greater than one standard deviation above the mean. (2)

1.1.3 Calculate the maximum total amount of data that Sam must use for the remainder of the month if she wishes for the overall mean of April to be 80% of the mean for the first 15 days. (3)

1.2 The wind speed (in km per hour) and temperature (in °C) for a certain town were recorded at 16:00 for a period of 10 days. The information is shown in the table below.

WIND SPEED IN km/h (x)	2	6	15	20	25	17	11	24	13	22
TEMPERATURE IN °C (y)	28	26	22	22	16	20	24	19	26	19

1.2.1 Determine the equation of the least squares regression line for the data. (3)

1.2.2 Predict the temperature at 16:00 if, on a certain day, the wind speed of this town was 9 km per hour. (2)

1.2.3 Interpret the value of b in the context of the data. (1)
[14]

QUESTION 2

The number of days that Grade 8 learners were absent at a certain high school during a year was recorded. This information is represented in the table below.

NUMBER OF DAYS ABSENT	NUMBER OF LEARNERS
$0 \leq x < 5$	34
$5 \leq x < 10$	45
$10 \leq x < 15$	98
$15 \leq x < 20$	43
$20 \leq x < 25$	7
$25 \leq x < 30$	3

2.1 Write down the modal class for the data. (1)

2.2 How many learners were absent from school for less than 15 days? (1)

2.3 How many Grade 8 learners are at the school? (1)

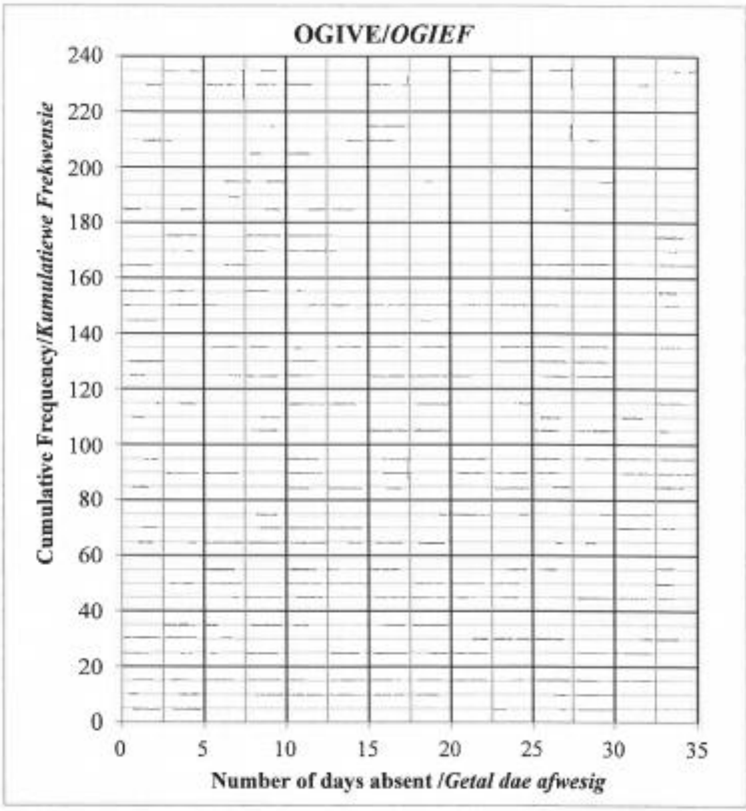
2.4 Draw a cumulative frequency graph (ogive) to represent the data above on the grid provided in the ANSWER BOOK. (4)

2.5 Use the cumulative frequency graph to determine the median number of days the Grade 8 learners were absent. (2)

[9]

ANSWER QUESTION 2.4 HERE:

2.4



(4)

(May/June 2019)

QUESTION 2

Learners who scored a mark below 50% in a Mathematics test were selected to use a computer-based programme as part of an intervention strategy. On completing the programme, these learners wrote a second test to determine the effectiveness of the intervention strategy. The mark (as a percentage) scored by 15 of these learners in both tests is given in the table below.

LEARNER	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15
TEST 1 (%)	10	18	23	24	27	34	34	36	37	39	40	44	45	48	49
TEST 2 (%)	33	21	32	20	58	43	49	48	41	55	50	45	62	68	60

2.1 Determine the equation of the least squares regression line. (3)

2.2 A learner's mark in the first test was 15 out of a maximum of 50 marks.

2.2.1 Write down the learner's mark for this test as a percentage. (1)

2.2.2 Predict the learner's mark for the second test. Give your answer to the nearest integer. (2)

2.3 For the 15 learners above, the mean mark of the second test is 45,67% and the standard deviation is 13,88%. The teacher discovered that he forgot to add the marks of the last question to the total mark of each of these learners. All the learners scored full marks in the last question. When the marks of the last question are added, the new mean mark is 50,67%.

2.3.1 What is the standard deviation after the marks for the last question are added to each learner's total? (2)

2.3.2 What is the total mark of the last question? (2)

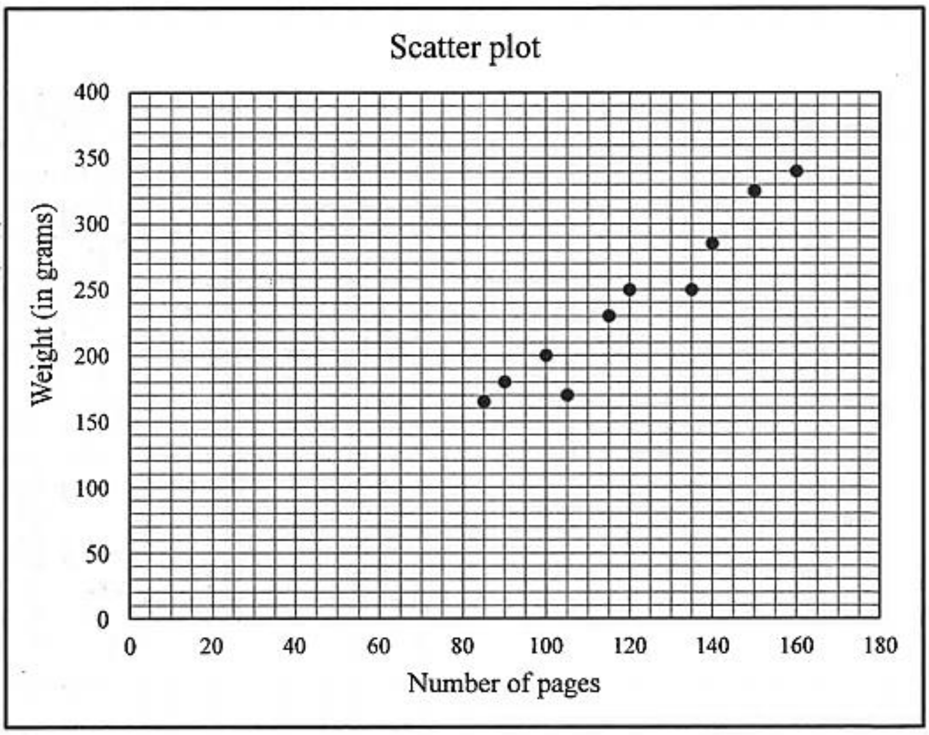
[10]

(May/June 2024)

QUESTION 1

The number of pages in ten A4 books and their corresponding weights (in grams) are given in the table below. The data is also represented in the scatter plot.

Number of pages (x)	85	150	100	120	90	140	135	105	115	160
Weight (in grams) (y)	165	325	200	250	180	285	250	170	230	340



1.1 Determine the equation of the least squares regression line. (3)

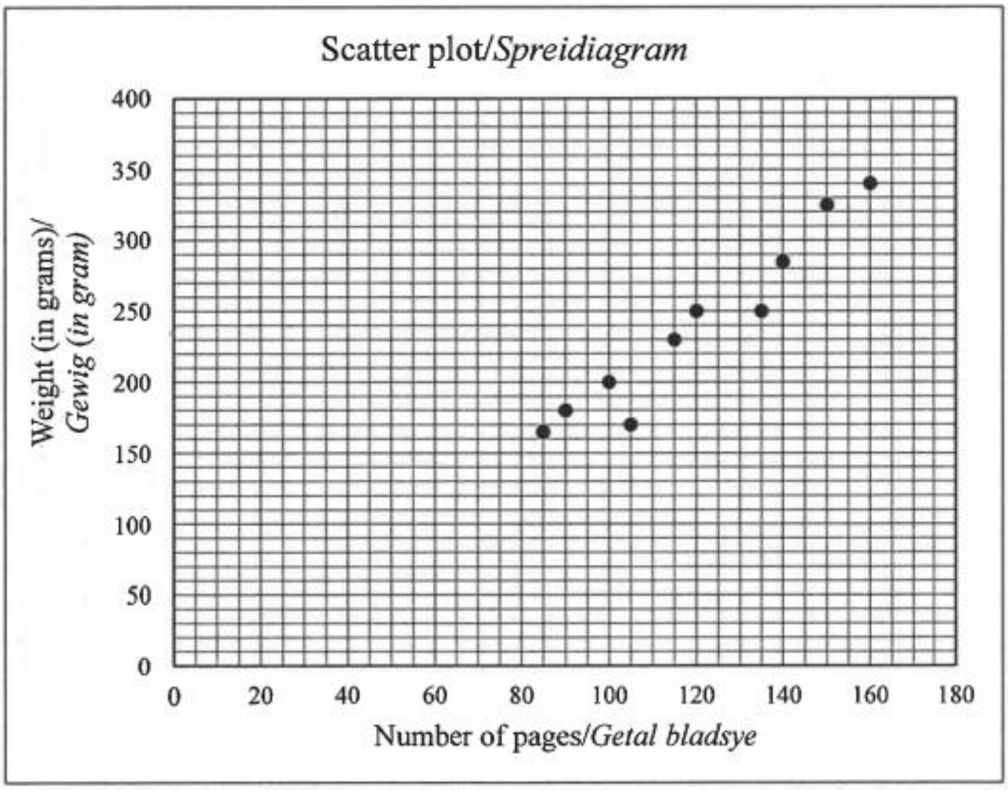
1.2 Draw the least squares regression line on the scatter plot in the ANSWER BOOK. (2)

1.3 Predict the weight of an A4 book that has 110 pages. (2)

1.4 Calculate the percentage weight increase between a book with 110 pages and a book with 130 pages. (3)
[10]

ANSWER QUESTION 1.2 HERE:

1.2



QUESTION 2

Fifty athletes need to access suitable training facilities. The table below shows the distances, in km, that they need to travel to obtain access to suitable training facilities.

DISTANCE (x km)	NUMBER OF ATHLETES
$0 \leq x < 5$	3
$5 \leq x < 10$	7
$10 \leq x < 15$	20
$15 \leq x < 20$	12
$20 \leq x < 25$	5
$25 \leq x < 30$	3

2.1 Complete the cumulative frequency column provided in the table in the ANSWER BOOK. (2)

2.2 On the grid provided in the ANSWER BOOK, draw a cumulative frequency graph (ogive) to represent the above data. (3)

2.3 Calculate the interquartile range (IQR) of the above data. (2)

2.4 The families of 4 of the athletes above who stay between 15 and 20 km from a suitable training facility, decide to move 10 kilometres closer to the facility. In which interval will the number of athletes increase? (1)

2.5 Calculate the estimated mean distance that the fifty athletes need to travel after the 4 families have moved 10 kilometres closer to the facility. Clearly show ALL working. (3)

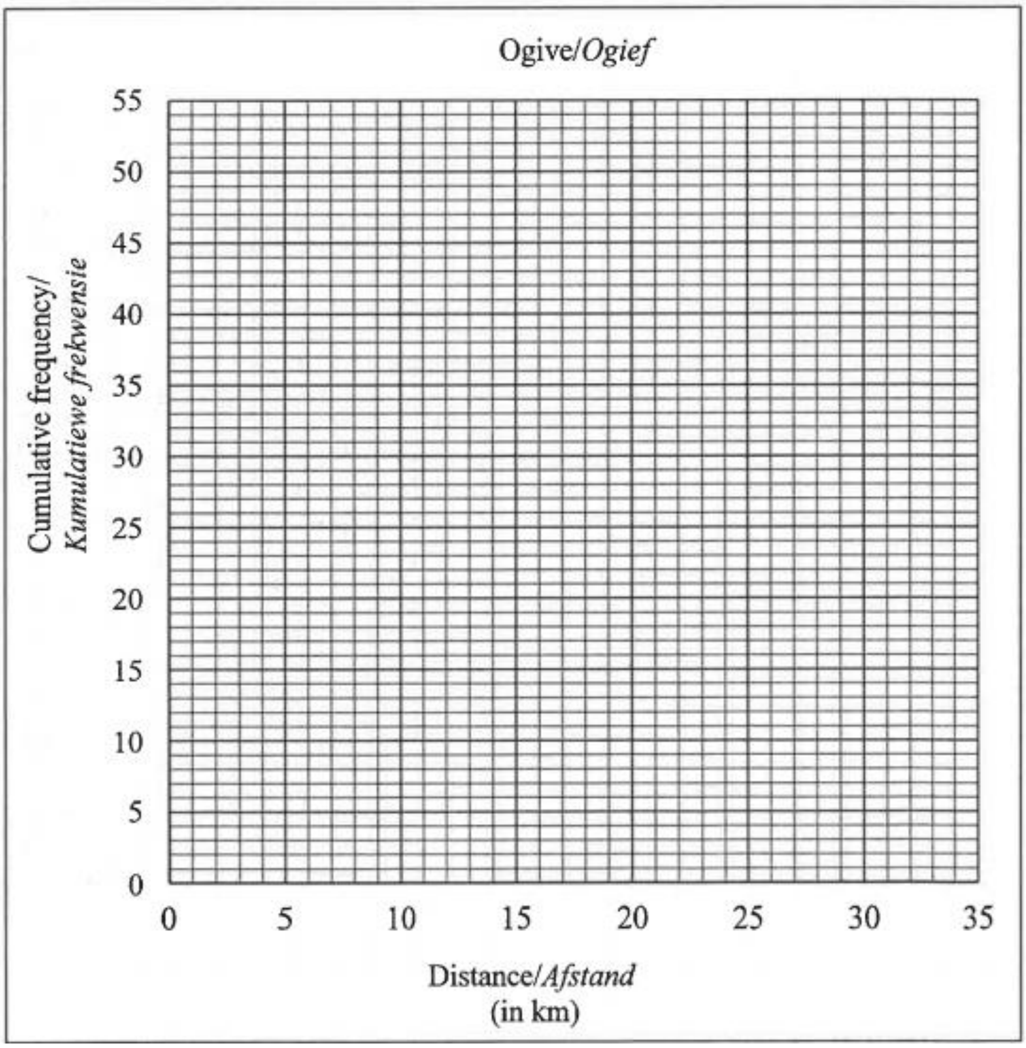
[11].

ANSWER QUESTION 2.1 AND 2.2 HERE:

2.1

Distance/*Afstand* (x km)	Number of athletes/ *Getal atlete*	Cumulative frequency/ *Kumulatiewe frekwensie*
$0 \leq x < 5$	3	
$5 \leq x < 10$	7	
$10 \leq x < 15$	20	
$15 \leq x < 20$	12	
$20 \leq x < 25$	5	
$25 \leq x < 30$	3	

2.2



(May/June 2023)

QUESTION 1

1.1 The owner of a small company wishes to establish whether advertising in a regional newspaper is effective. The table below shows the amount spent on advertising and the corresponding sales figures for the last 9 years.

Amount spent on advertising (in rands) (x)	21 300	23 700	24 800	30 540	24 100	40 680	22 400	35 250	29 110
Sales (in rands) (y)	311 500	326 700	349 200	470 000	316 100	564 200	314 000	487 300	392 900

1.1.1 Determine the equation of the least squares regression line for the data. (3)

1.1.2 Predict the sales for a year in which the company will spend R28 500 on advertising. (2)

1.1.3 Write down the correlation coefficient of the data. (1)

1.1.4 Describe the association between the amount spent on advertising in the regional newspaper and the sales of this company. (1)

1.2 The profit that the small company made over the same 9 years is given in the table below.

Profit (in rands)	110 750	107 376	152 338	244 480	144 021	275 994	121 900	207 636	187 700

1.2.1 Calculate the mean profit made over the 9 years. (2)

1.2.2 Write down the standard deviation for the data. (1)

1.2.3 Determine the number of years in which the company made a profit that was greater than one standard deviation above the mean. (2)

[12]

QUESTION 2

The ages of the people who attended a music concert was summarised in the table below.

AGE	NUMBER OF PEOPLE
$5 < x \leq 15$	20
$15 < x \leq 25$	25
$25 < x \leq 35$	60
$35 < x \leq 45$	90
$45 < x \leq 55$	55
$55 < x \leq 65$	40
$65 < x \leq 75$	30

2.1 Write down the modal class of the data. (1)

2.2 How many people attended the music concert? (1)

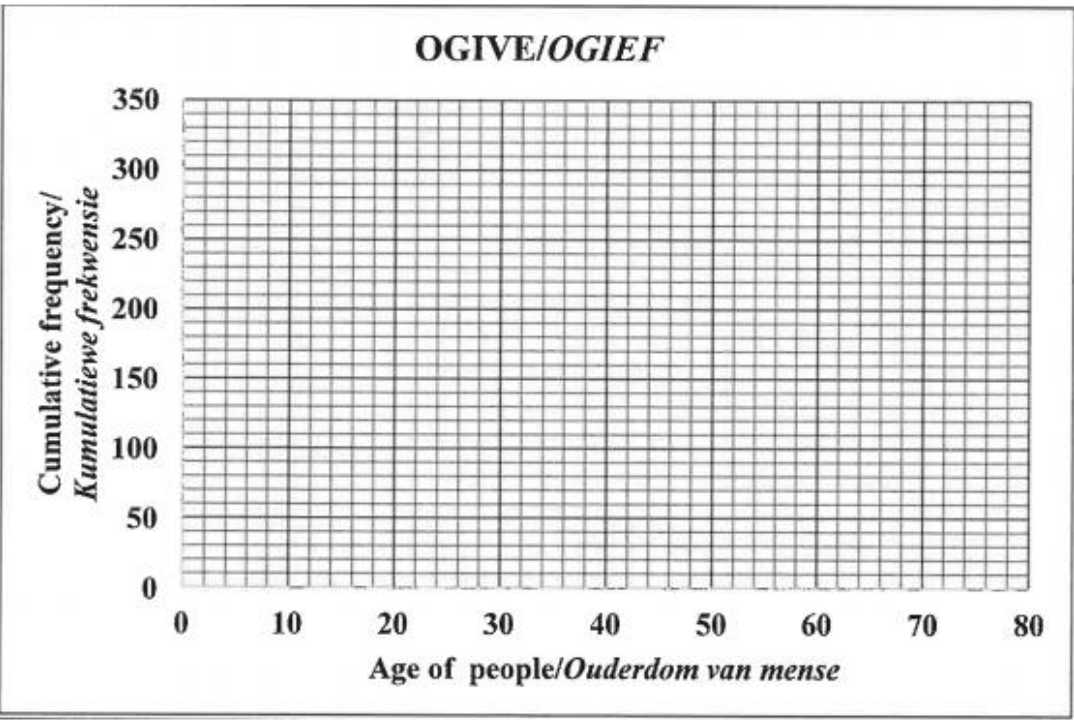
2.3 On the grid provided in the ANSWER BOOK, draw a cumulative frequency graph (ogive) to represent the above data. (4)

2.4 Use the cumulative frequency graph to determine the median age of the people who attended the music concert. (2)

[8]

ANSWER QUESTION 2.3 HERE:

2.3



(May/June 2022)

QUESTION 1

The table below shows the mass (in kg) of the school bags of 80 learners.

MASS (in kg)	FREQUENCY
$5 < m \leq 7$	6
$7 < m \leq 9$	18
$9 < m \leq 11$	21
$11 < m \leq 13$	19
$13 < m \leq 15$	11
$15 < m \leq 17$	4
$17 < m \leq 19$	1

1.1 Write down the modal class of the data. (1)

1.2 Complete the cumulative frequency column in the table in the ANSWER BOOK. (2)

1.3 Draw a cumulative frequency graph (ogive) for the given data on the grid provided in the ANSWER BOOK. (3)

1.4 Use the graph to determine the median mass for this data. (2)

1.5 The international guideline for the mass of a school bag is that it should not exceed 10% of a learner's body mass.

1.5.1 Calculate the estimated mean mass of the school bags. (2)

1.5.2 The mean mass of this group of learners was found to be 80 kg. On average, are these school bags satisfying the international guideline with regard to mass? Motivate your answer. (2)

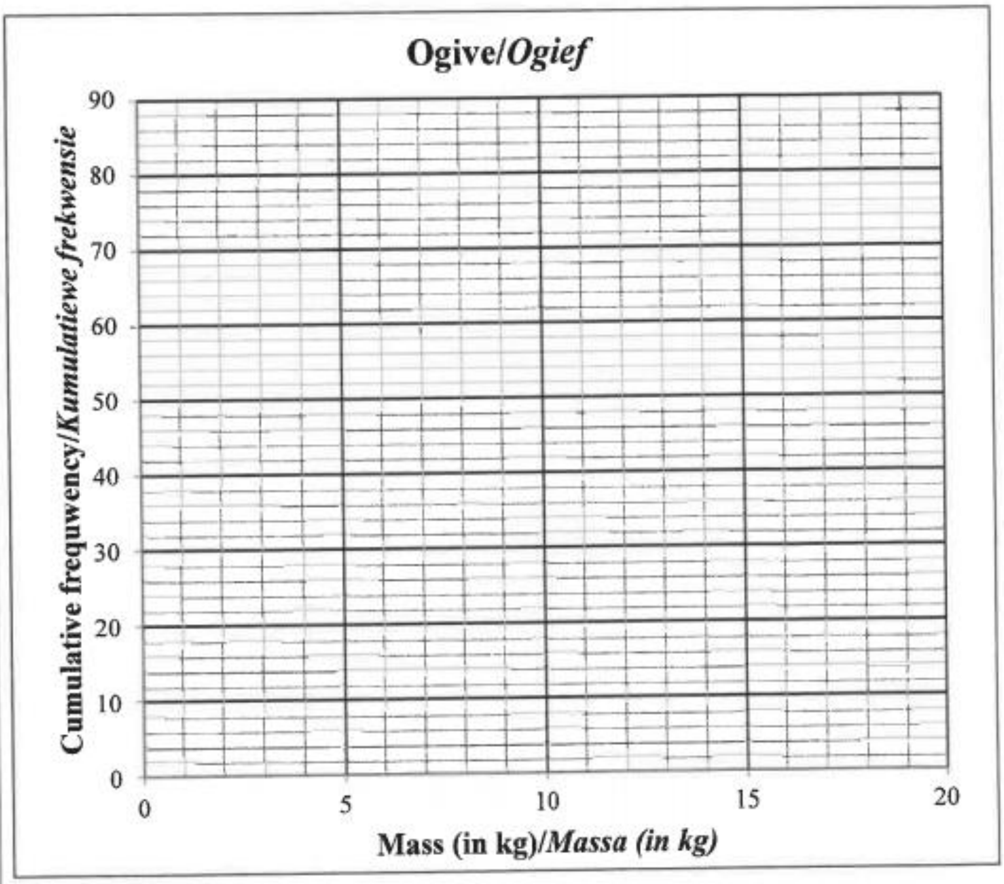
[12]

ANSWER QUESTION 1.2 AND 1.3 HERE:

1.2

MASS (in kg)/ *MASSA (in kg)*	FREQUENCY/ *FREKWENSIE*	CUMULATIVE FREQUENCY/ *KUMULATIEWE FREKWENSIE*
$5 < m \leq 7$	6	
$7 < m \leq 9$	18	
$9 < m \leq 11$	21	
$11 < m \leq 13$	19	
$13 < m \leq 15$	11	
$15 < m \leq 17$	4	
$17 < m \leq 19$	1	

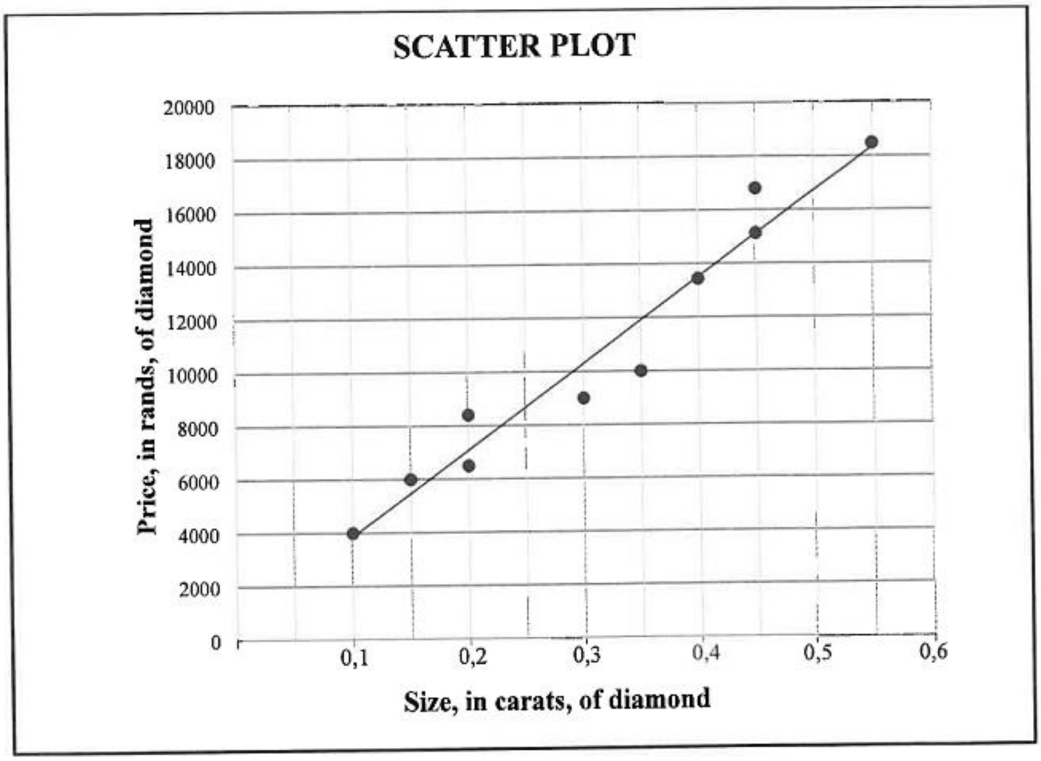
1.3



QUESTION 2

The table below shows the size (in carats) and the price (in rands) of 10 diamonds that were sold by a diamond trader. This information is also presented in the scatter plot below. The least squares regression line for the data is drawn.

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480



2.1 Determine the equation of the least squares regression line for the data. (3)

2.2 If the trader sold a diamond that was 0,25 carats in size, predict the selling price of this diamond in rands. (2)

2.3 Calculate the average price increase per 0,05 carat of the diamonds. (2)

2.4 It was later found that the selling price of the 0,35 carat diamond was recorded incorrectly. The correct price is R11 500. When this correction is made to the data set, the correlation between the size and price of these diamonds gets stronger. Explain the reason for this by referring to the given scatter plot. (1)

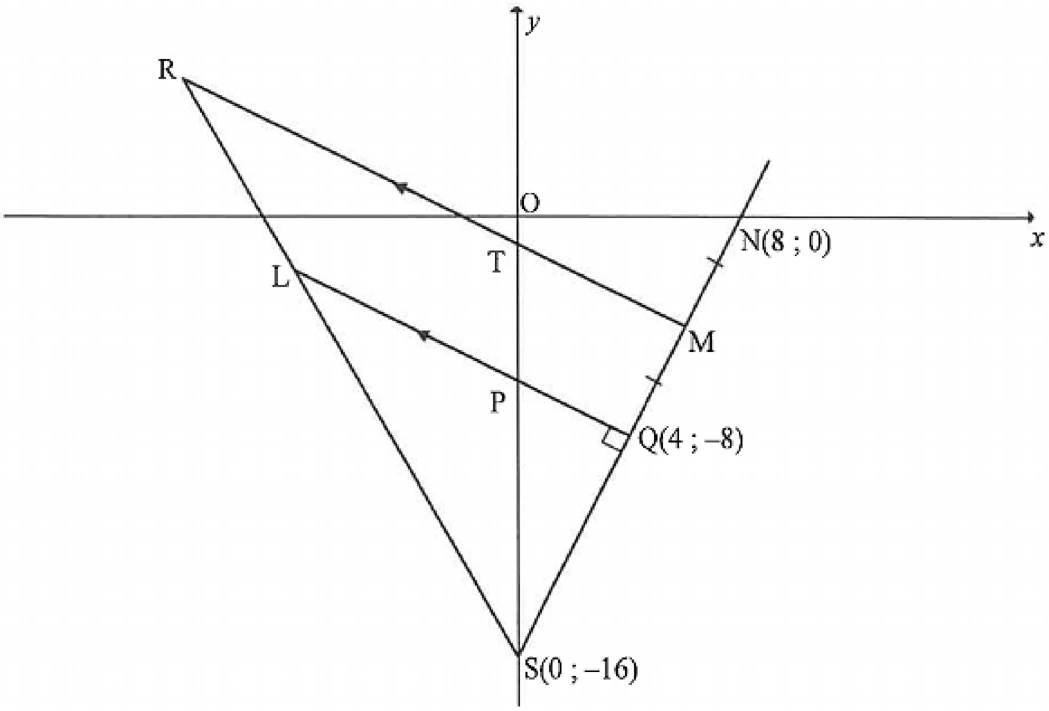
[8]

Analytical Geometry

(May/June 2021)

QUESTION 3

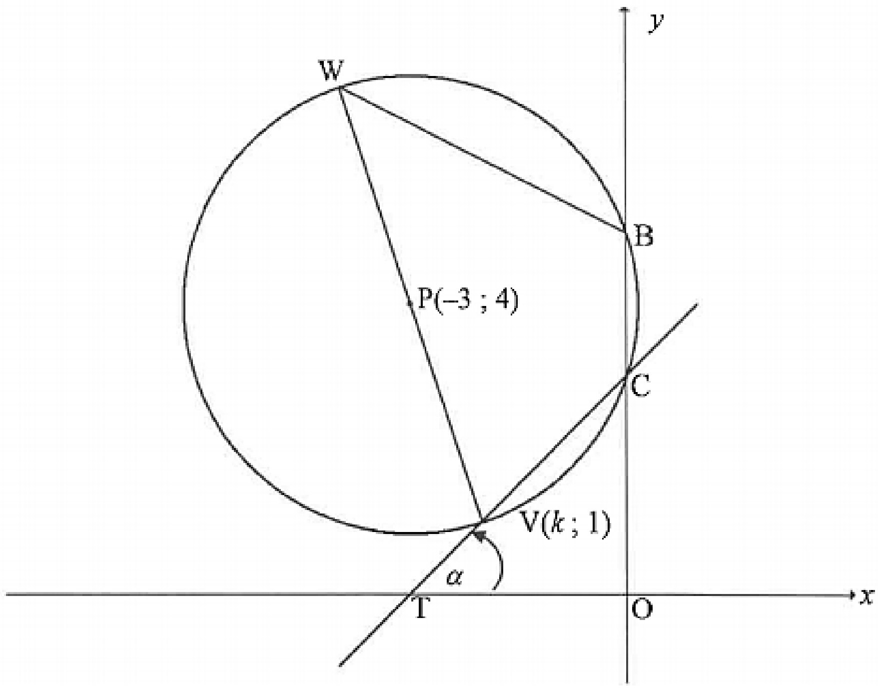
In the diagram, S(0 ; –16), L and Q(4 ; –8) are the vertices of ΔSLQ having LQ perpendicular to SQ. SL and SQ are produced to points R and M respectively such that RM || LQ. SM produced cuts the x-axis at N(8 ; 0). QM = MN. T and P are the y-intercepts of RM and LQ respectively.



3.1	Calculate the coordinates of M.	(2)
3.2	Calculate the gradient of NS.	(2)
3.3	Show that the equation of line LQ is $y=-\frac{1}{2}x-6$.	(3)
3.4	Determine the equation of a circle having centre at O, the origin, and also passing through S.	(2)
3.5	Calculate the coordinates of T.	(3)
3.6	Determine $\frac{LS}{RS}$.	(3)
3.7	Calculate the area of PTMQ.	(4)
		[19]

QUESTION 4

In the diagram, $P(-3\ ;\ 4)$ is the centre of the circle. $V(k\ ;\ 1)$ and W are the endpoints of a diameter. The circle intersects the y-axis at B and C. BCVW is a cyclic quadrilateral. CV is produced to intersect the x-axis at T. $O\hat{T}C = \alpha$.



4.1 The radius of the circle is $\sqrt{10}$. Calculate the value of k if point V is to the right of point P. Clearly show ALL calculations. (5)

4.2 The equation of the circle is given as $x^2+6x+y^2-8y+15=0$. Calculate the length of BC. (4)

4.3 If $k=-2$, calculate the size of:

4.3.1 α (3)

4.3.2 $V\hat{W}B$ (2)

4.4 A new circle is obtained when the given circle is reflected about the line $y=1$.

Determine the:

4.4.1 Coordinates of Q, the centre of the new circle (2)

4.4.2 Equation of the new circle in the form $(x-a)^2+(y-b)^2=r^2$ (2)

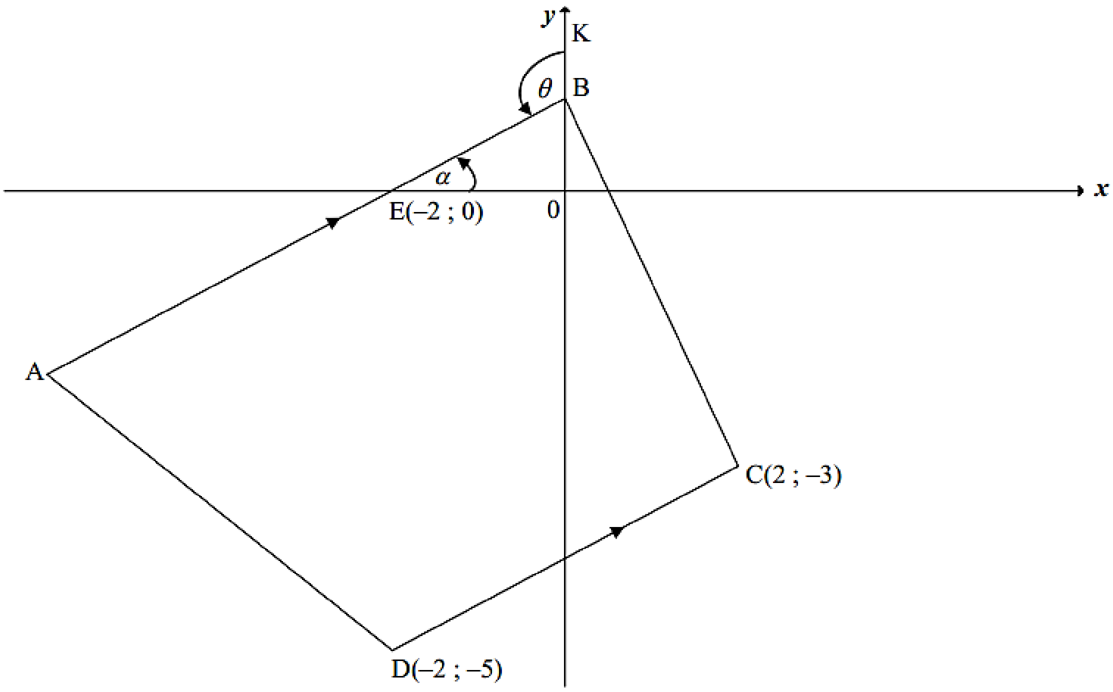
4.4.3 Equations of the lines drawn parallel to the y-axis and passing through the points of intersection of the two circles (2)

[20]

(May/June 2019)

QUESTION 3

In the diagram, A, B, C(2 ; –3) and D(–2 ; –5) are vertices of a trapezium with AB || DC. E(–2 ; 0) is the x-intercept of AB. The inclination of AB is α. K lies on the y-axis and $K\hat{B}E = \theta$.



3.1 Determine:

3.1.1 The midpoint of EC (2)

3.1.2 The gradient of DC (2)

3.1.3 The equation of AB in the form $y = mx + c$ (3)

3.1.4 The size of θ (3)

3.2 Prove that AB $\perp$ BC. (3)

3.3 The points E, B and C lie on the circumference of a circle. Determine:

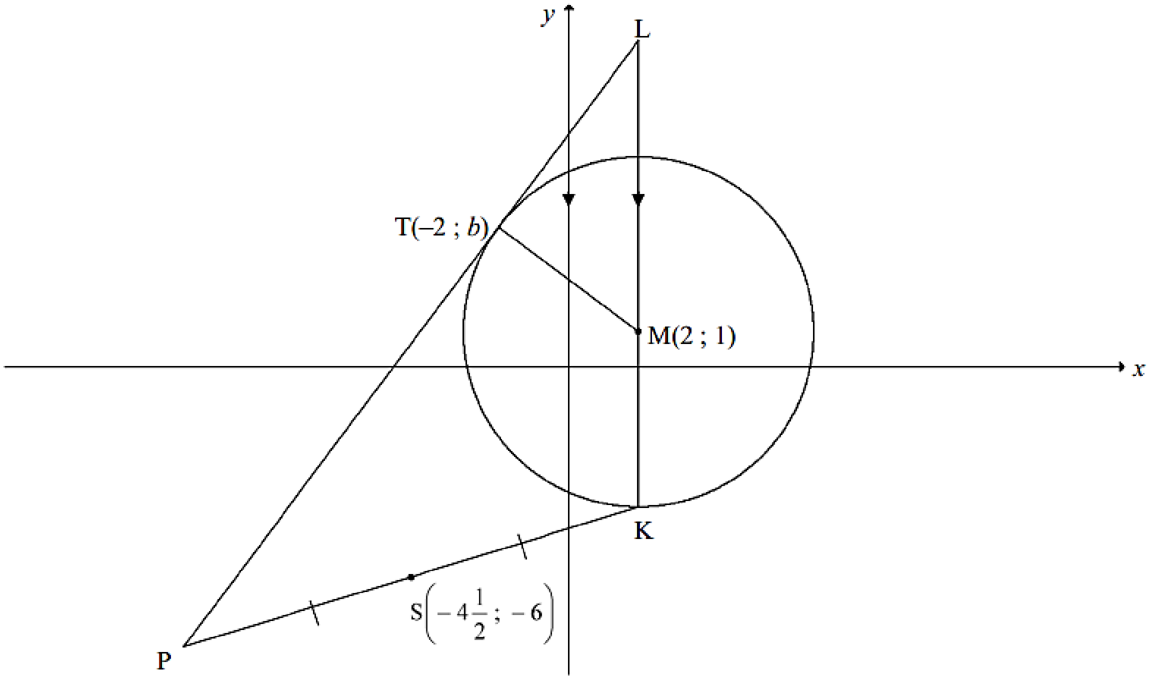
3.3.1 The centre of the circle (1)

3.3.2 The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (4)

[18]

QUESTION 4

In the diagram, the circle is centred at M(2 ; 1). Radius KM is produced to L, a point outside the circle, such that KML ∥ y-axis. LTP is a tangent to the circle at T(–2 ; b). $S\left(-4\frac{1}{2};-6\right)$ is the midpoint of PK.



4.1 Given that the radius of the circle is 5 units, show that $b=4$. (4)

4.2 Determine:

4.2.1 The coordinates of K (2)

4.2.2 The equation of the tangent LTP in the form $y = mx + c$ (4)

4.2.3 The area of ΔLPK (7)

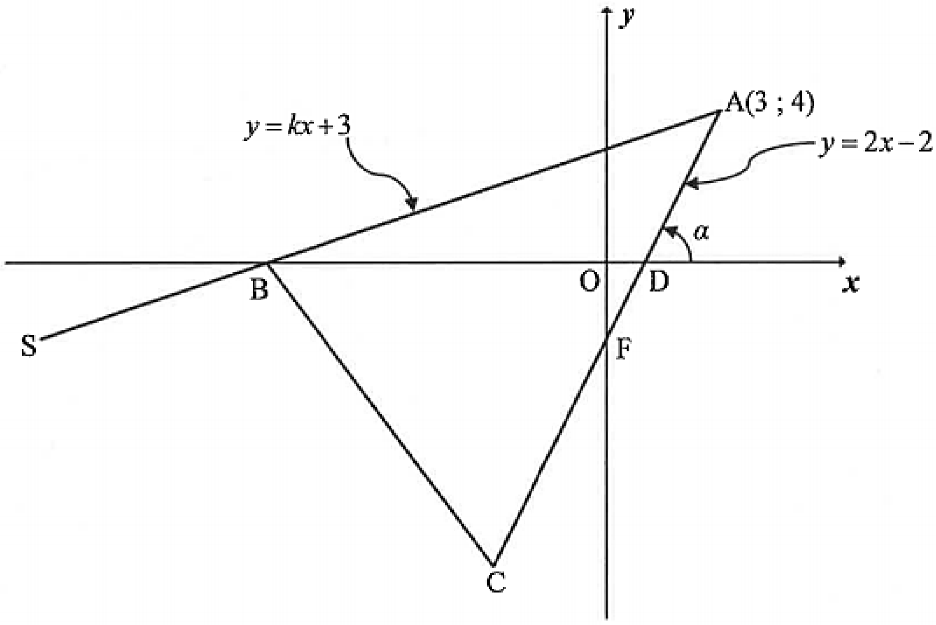
4.3 Another circle with equation $(x-2)^2+(y-n)^2=25$ is drawn. Determine, with an explanation, the value(s) of n for which the two circles will touch each other externally. (4)

[21]

(May/June 2024)

QUESTION 3

In the diagram, A(3 ; 4), B and C are vertices of ΔABC. AB is produced to S. D and F are the x- and y-intercepts of AC respectively. F is the midpoint of AC and the angle of inclination of AC is α. The equation of AB is $y = kx + 3$ and the equation of AC is $y = 2x - 2$.



3.1 Show that $k = \frac{1}{3}$. (1)

3.2 Calculate the coordinates of B, the x-intercept of line AS. (2)

3.3 Calculate the coordinates of C. (4)

3.4 Determine the equation of the line parallel to BC and passing through S(–15 ; –2). Write your answer in the form $y = mx + c$. (5)

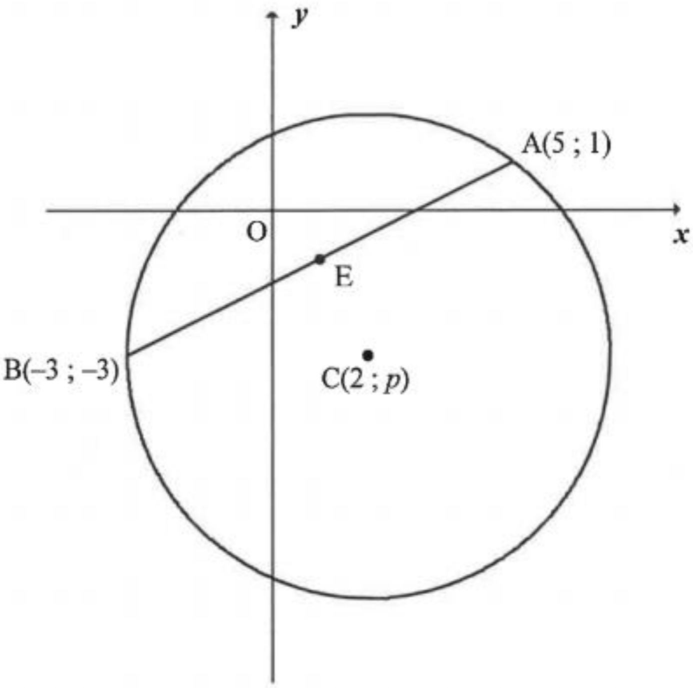
3.5 Calculate the size of $B\hat{A}C$. (5)

3.6 If it is further given that the length of AC is $6\sqrt{5}$ units, calculate the value of $\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC}$. (5)

[22]

QUESTION 4

In the diagram, the circle centred at $C(2\,;p)$ is drawn. $A(5\,;1)$ and $B(-3;-3)$ are points on the circle. E is the midpoint of AB.

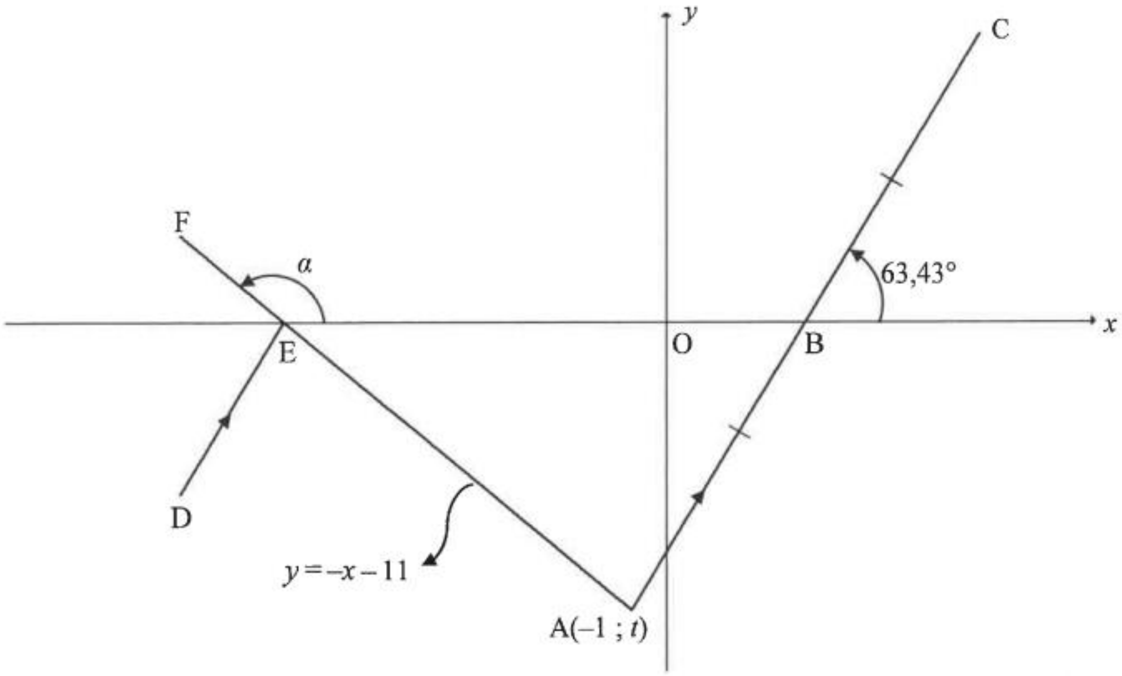


4.1	Calculate the coordinates of E, the midpoint of AB.	(2)
4.2	Calculate the length of AB. Leave your answer in surd form.	(1)
4.3	Determine the equation of the perpendicular bisector of AB in the form $y = mx + c$.	(4)
4.4	Show that $p = -3$.	(1)
4.5	Show, by calculation, that the equation of the circle is $x^2 + y^2 - 4x + 6y - 12 = 0$	(4)
4.6	Calculate the values of t for which the straight line $y = tx + 8$ will not intersect the circle.	(6)
		[18]

(May/June 2023)

QUESTION 3

In the diagram, the equation of line AF is $y = -x - 11$. B, a point on the x-axis, is the midpoint of the straight line joining A$(-1\ ;\ t)$ and C. The angles of inclination of AF and AC are α and $63{,}43°$ respectively. AF cuts the x-axis in E. D is a point such that DE $\parallel$ AC.



3.1 Calculate the:

3.1.1 Value of t (2)

3.1.2 Size of α (2)

3.1.3 Gradient of AC, to the nearest whole number (2)

3.2 Determine the equation of AC in the form $y = mx + k$. (2)

3.3 Calculate the:

3.3.1 Coordinates of C (3)

3.3.2 Size of $\mathrm{F\hat{E}D}$ (3)

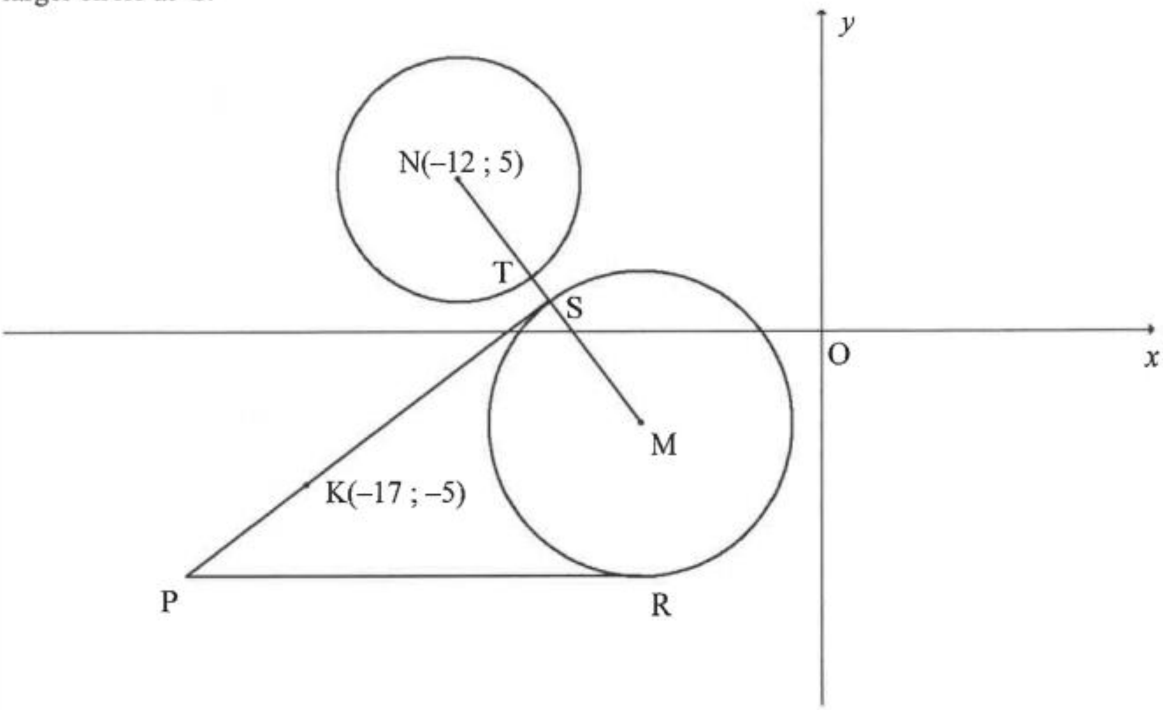
3.4 G is a point such that EAGC, in that order, is a parallelogram.

Determine the equation of a circle centred at G and passing through the point B. Write your answer in the form $(x-a)^2 + (y-b)^2 = r^2$. (4)

[18]

QUESTION 4

In the diagram, the equation of the circle centred at N(–12 ; 5) is $x^2 + y^2 + 24x - 10y + 153 = 0$. The equation of the circle centred at M is $(x+6)^2 + (y+3)^2 = 25$. PS and PR are tangents to the circle centred at M at S and R respectively. PR is parallel to the x-axis. K(–17 ; –5) is a point on PS. The straight line joining N and M cuts the smaller circle at T and the larger circle at S.



4.1 Write down the coordinates of M. (2)

4.2 Calculate the:

4.2.1 Length of the radius of the smaller circle (2)

4.2.2 Length of TS (4)

4.3 Determine the equation of the tangent:

4.3.1 PR (2)

4.3.2 PS, in the form $y = mx + c$ (5)

4.4 Quadrilateral PSMR is drawn. Calculate the:

4.4.1 Perimeter of PSMR (5)

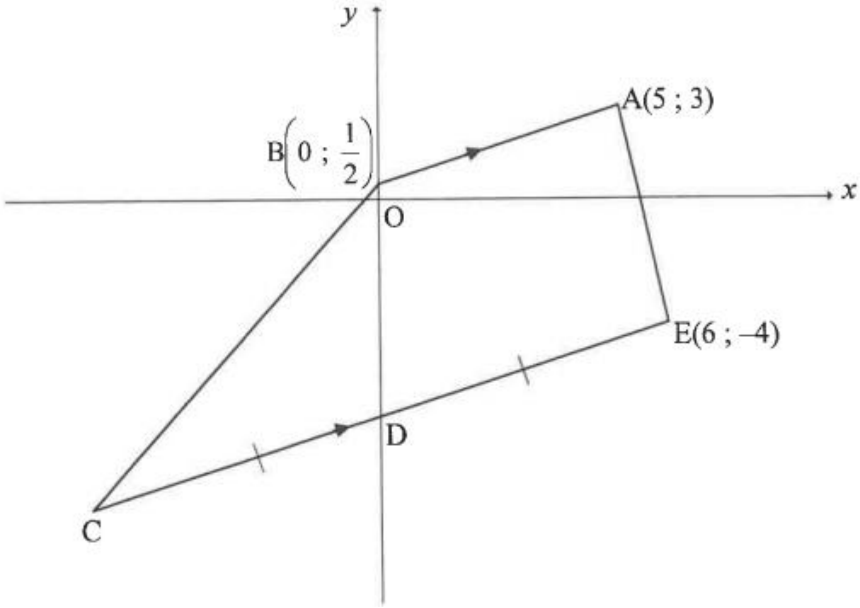
4.4.2 Ratio of $\dfrac{\text{area of } \Delta\text{NPS}}{\text{area of quadrilateral PSMR}}$ (2)

[22]

(May/June 2022)

QUESTION 3

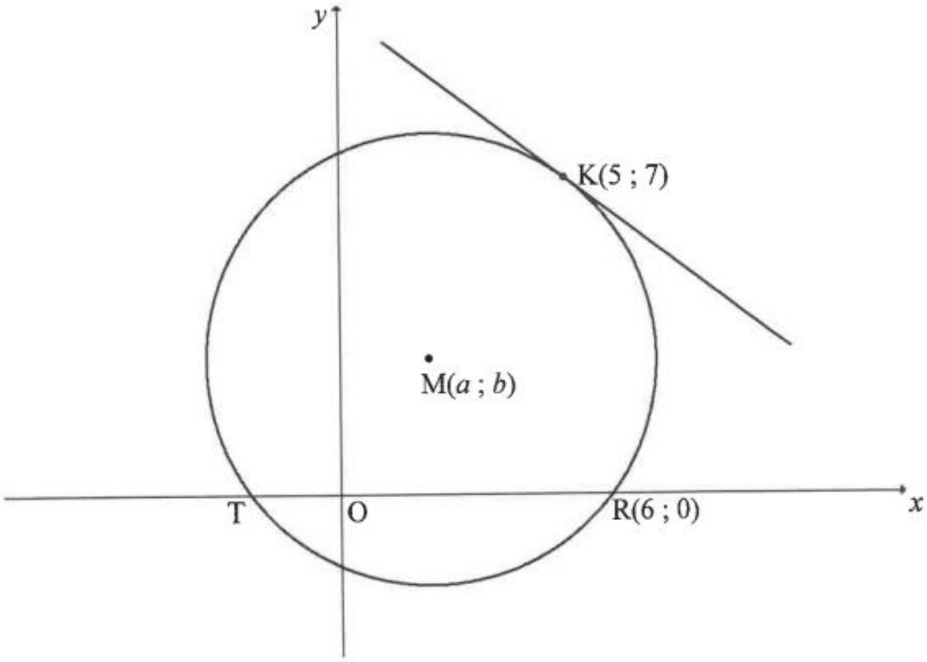
In the diagram, A(5 ; 3), $B\left(0\,;\,\frac{1}{2}\right)$, C and E(6 ; –4) are the vertices of a trapezium having BA || CE. D is the y-intercept of CE and CD = DE.



3.1	Calculate the gradient of AB.		(2)
3.2	Determine the equation of CE in the form $y = mx + c$.		(3)
3.3	Calculate the:		
	3.3.1	Coordinates of C	(3)
	3.3.2	Area of quadrilateral ABCD	(4)
3.4	If point K is the reflection of E in the y-axis:		
	3.4.1	Write down the coordinates of K	(2)
	3.4.2	Calculate the:	
		(a) Perimeter of ΔKEC	(4)
		(b) Size of $K\hat{C}E$	(3)
			[21]

QUESTION 4

In the diagram, the circle centred at $M(a\ ;\ b)$ is drawn. T and $R(6\ ;\ 0)$ are the x-intercepts of the circle. A tangent is drawn to the circle at $K(5\ ;\ 7)$.



4.1 M is a point on the line $y = x + 1$.

4.1.1 Write b in terms of a. (1)

4.1.2 Calculate the coordinates of M. (5)

4.2 If the coordinates of M are $(2\ ;\ 3)$, calculate the length of:

4.2.1 The radius of the circle (2)

4.2.2 TR (2)

4.3 Determine the equation of the tangent to the circle at K. Write your answer in the form $y = mx + c$. (5)

4.4 A horizontal line is drawn as a tangent to the circle M at the point $N(c\ ;\ d)$, where $d < 0$.

4.4.1 Write down the coordinates of N. (2)

4.4.2 Determine the equation of the circle centred at N and passing through T. Write your answer in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

[20]

Trigonometry

(May/June 2024)

QUESTION 5

5.1 If $\sin 40° = p$, write EACH of the following in terms of p.

5.1.1 $\sin 220°$ (2)

5.1.2 $\cos^2 50°$ (2)

5.1.3 $\cos(-80°)$ (3)

5.2 Given: $\tan x(1-\cos^2 x)+\cos^2 x=\dfrac{(\sin x+\cos x)(1-\sin x\cos x)}{\cos x}$

5.2.1 Prove the above identity. (5)

5.2.2 For which values of x, in the interval $x \in [-180°;180°]$, will the identity be undefined? (3)

5.3 Given the expression: $\dfrac{\sin 150°+\cos^2 x-1}{2}$

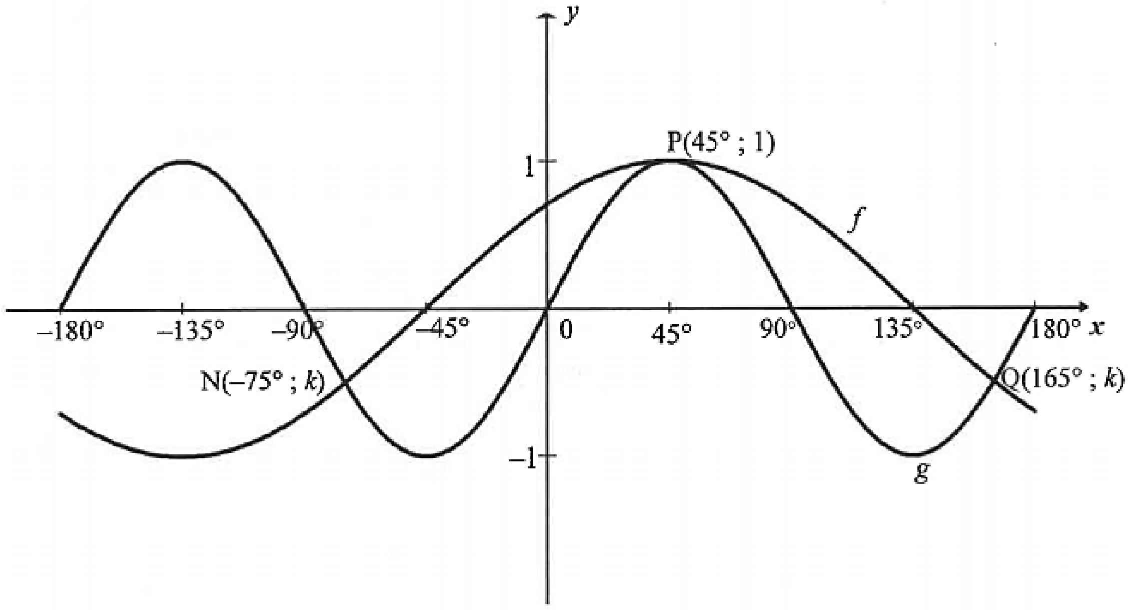
5.3.1 **Without using a calculator**, simplify the expression given above to a single trigonometric term in terms of $\cos 2x$. (6)

5.3.2 Hence, determine the general solution of $\dfrac{\sin 150°+\cos^2 x-1}{2}=\dfrac{1}{25}$ (5)

[26]

QUESTION 6

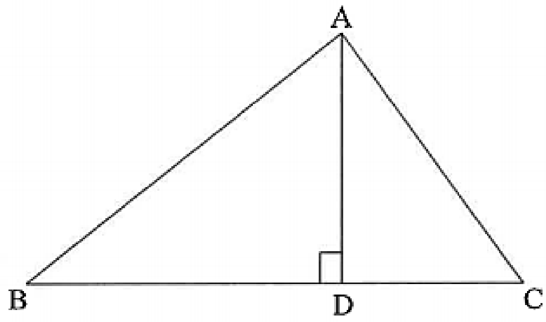
In the diagram, the graphs of $f(x)=\cos(x+a)$ and $g(x)=\sin 2x$ are drawn for the interval $x\in[-180°;180°]$. The graphs intersect at N(–75° ; k), P(45° ; 1) and Q(165° ; k). P is also a turning point of both graphs.



6.1	Write down the period of f.	(1)
6.2	Write down the amplitude of g.	(1)
6.3	Write down the value of a.	(1)
6.4	Calculate the value of k, the y-coordinate of N and Q, **without the use of a calculator.**	(2)
6.5	Calculate the value of x if $g(x+60°)=f(x+60°)$ and $x\in[-45°;0°]$.	(1)
6.6	**Without using a calculator**, determine the number of solutions the equation $\sqrt{2}\sin 2x=\sin x+\cos x$ has in the interval $x\in[-90°;90°]$. Clearly show ALL working.	(4)
		[10]

QUESTION 7

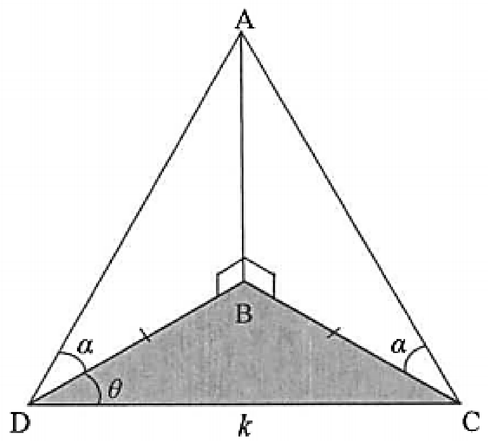
7.1 In the diagram, ΔABC is drawn. AD is drawn such that $AD \perp BC$.



7.1.1 Use the diagram above to determine AD in terms of $\sin\hat{B}$ (2)

7.1.2 Hence, prove that the area of $\Delta ABC = \frac{1}{2}(BC)(AB)\sin\hat{B}$ (1)

7.2 In the diagram, points B, C and D lie in the same horizontal plane. $A\hat{D}B = A\hat{C}B = \alpha$, $C\hat{D}B = \theta$ and $DC = k$ units. $BD = BC$.



7.2.1 Prove that $AD = AC$ (2)

7.2.2 Prove that $BD = \dfrac{k}{2\cos\theta}$ (3)

7.2.3 Determine the area of ΔBCD in terms of k and a single trigonometric ratio of θ. (3)

[11]

(May/June 2023)

QUESTION 5

5.1 **Without using a calculator**, simplify the following expression to a single trigonometry ratio:

$$\frac{1-\sin(-\theta)\cos(90°+\theta)}{\cos(\theta-360°)}$$ (5)

5.2 Given that $\cos 20° = p$

Without using a calculator, write EACH of the following in terms p:

5.2.1 $\cos 200°$ (2)

5.2.2 $\sin(-70°)$ (2)

5.2.3 $\sin 10°$ (3)

5.3 Determine, **without using a calculator**, the value of:

$$\cos(A+55°)\cos(A+10°)+\sin(A+55°)\sin(A+10°)$$ (3)

5.4 Consider: $\dfrac{\cos 2x+\sin 2x-\cos^2 x}{\sin x-2\cos x}=-\sin x$

5.4.1 Prove the above identity. (3)

5.4.2 Determine the value of $\dfrac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin^2 x+6\sin x\cos x}$ (3)

5.5 Given: $3\tan 4x=-2\cos 4x$

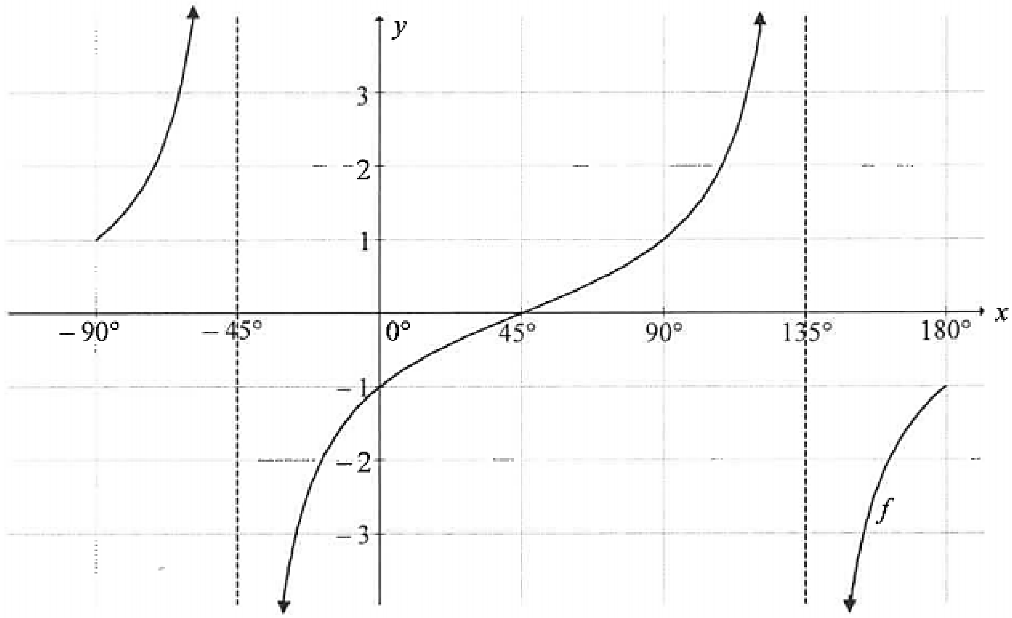
5.5.1 **Without using a calculator**, show that $\sin 4x=-0{,}5$ is the only solution to the above equation. (4)

5.5.2 Hence, determine the general solution of x in the equation $3\tan 4x=-2\cos 4x$ (3)

[28]

QUESTION 6

In the diagram below, the graph of $f(x)=\tan(x-45°)$ is drawn for $x\in[-90°\,;180°]$.



6.1 Write down the period of f. (1)

6.2 Draw the graph of $g(x)=-\cos 2x$ for the interval $x\in[-90°\,;180°]$ on the grid given in the ANSWER BOOK. Show ALL intercepts with the axes, as well as the minimum and maximum points of the graph. (3)

6.3 Write down the range of g. (1)

6.4 The graph of g is shifted $45°$ to the left to form the graph of h. Determine the equation of h in its simplest form. (2)

6.5 Use the graph(s) to determine the values of x in the interval $x\in[-90°\,;\,90°]$ for which:

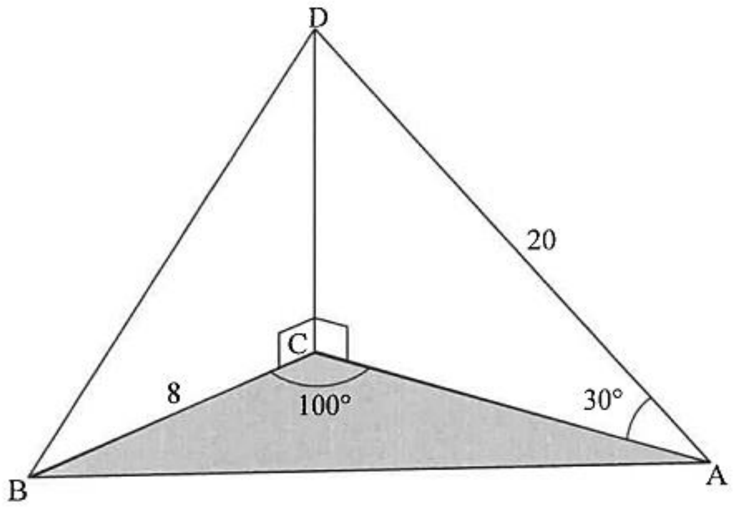
6.5.1 $f(x)>1$ (2)

6.5.2 $2\cos 2x-1>0$ (4)

[13]

QUESTION 7

In the diagram, A, B and C are points in the same horizontal plane. D is a point directly above C, that is $DC \perp AC$ and $DC \perp BC$. It is given that $A\hat{C}B = 100°$, $C\hat{A}D = 30°$, $AD = 20$ units and $BC = 8$ units.



7.1 Calculate the length of:

7.1.1 AC (2)

7.1.2 AB (3)

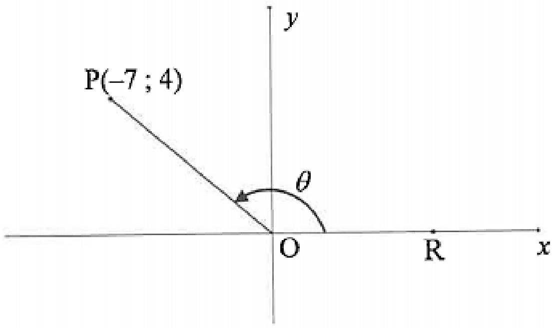
7.2 If it is further given that $A\hat{B}D = 73{,}4°$, calculate the size of $A\hat{D}B$. (3)

[8]

(May/June 2022)

QUESTION 5

5.1 In the diagram below, P(–7 ; 4) is a point in the Cartesian plane. R is a point on the positive x-axis such that obtuse $\hat{POR} = \theta$.



Calculate, **without using a calculator**, the:

5.1.1 Length OP (2)

5.1.2 Value of:

(a) $\tan\theta$ (1)

(b) $\cos(\theta - 180°)$ (2)

5.2 Determine the general solution of: $\sin x\cos x + \sin x = 3\cos^2 x + 3\cos x$ (7)

5.3 Given the identity: $\dfrac{\sin 3x}{1-\cos 3x} = \dfrac{1+\cos 3x}{\sin 3x}$

5.3.1 Prove the identity given above. (3)

5.3.2 Determine the values of x, in the interval $x \in [0° ; 60°]$, for which the identity will be undefined. (3)

[18]

QUESTION 6

6.1 **Without using a calculator**, simplify the following expression to a single trigonometric term:

$$\frac{\sin 10°}{\cos 440°} + \tan(360° - \theta).\sin 2\theta$$ (6)

6.2 Given: $\sin(60° + 2x) + \sin(60° - 2x)$

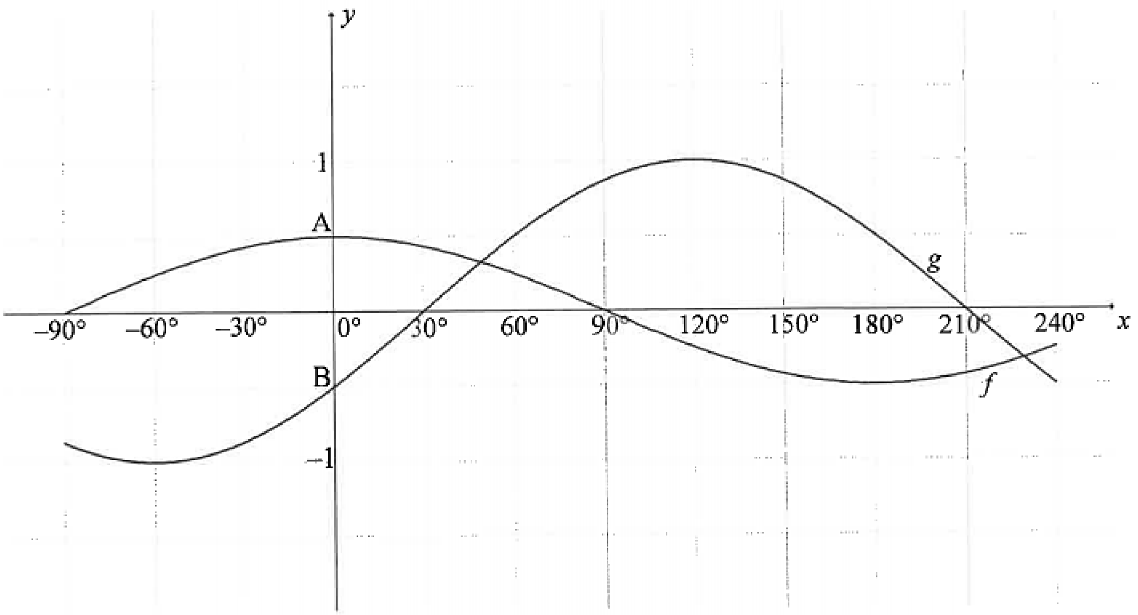
6.2.1 Calculate the value of k if $\sin(60° + 2x) + \sin(60° - 2x) = k\cos 2x$. (3)

6.2.2 If $\cos x = \sqrt{t}$, **without using a calculator**, determine the value of $\tan 60°[\sin(60° + 2x) + \sin(60° - 2x)]$ in terms of t. (3)

[12]

QUESTION 7

In the diagram below, the graphs of $f(x) = \frac{1}{2}\cos x$ and $g(x) = \sin(x - 30°)$ are drawn for the interval $x \in [-90° ; 240°]$. A and B are the y-intercepts of f and g respectively.



7.1 Determine the length of AB. (2)

7.2 Write down the range of $3f(x) + 2$. (2)

7.3 Read off from the graphs a value of x for which $g(x) - f(x) = \frac{\sqrt{3}}{2}$. (2)

7.4 For which values of x, in the interval $x \in [-90° ; 240°]$, will:

7.4.1 $f(x).g(x) > 0$ (2)

7.4.2 $g'(x - 5°) > 0$ (2)

[10]

QUESTION 8

FIGURE I shows a ramp leading to the entrance of a building. B, C and D lie on the same horizontal plane. The perpendicular height (AC) of the ramp is 0,5 m and the angle of elevation from B to A is 15°. The entrance of the building (AE) is 0,915 m wide.

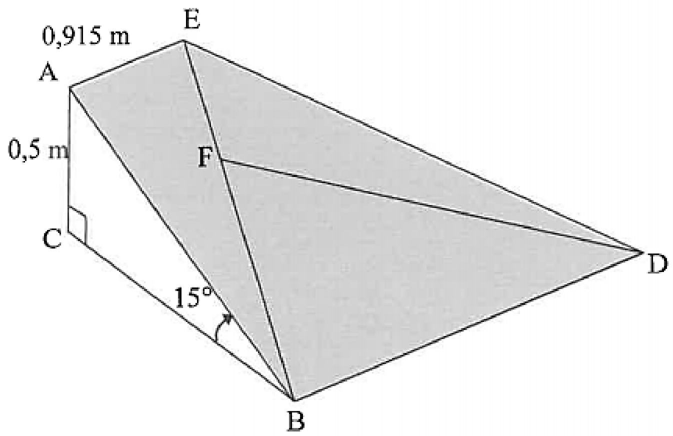


FIGURE I

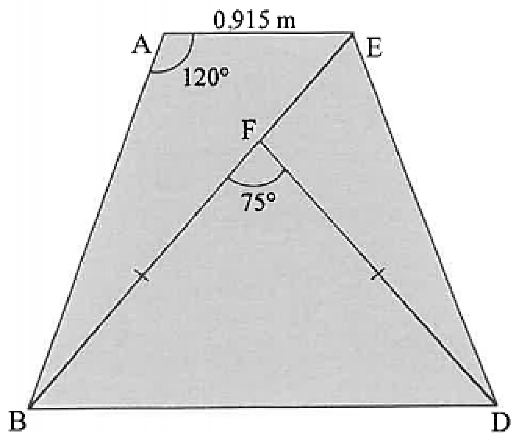


FIGURE II (top view)

8.1 Calculate the length of AB. (2)

8.2 Figure II shows the top view of the ramp. The area of the top of the ramp is divided into three triangles, as shown in the diagram.

If $B\hat{A}E = 120°$, calculate the length of BE. (3)

8.3 Calculate the area of ΔBFD if $B\hat{F}D = 75°$, $BF = FD$ and $BF = \frac{5}{7}BE$. (3)

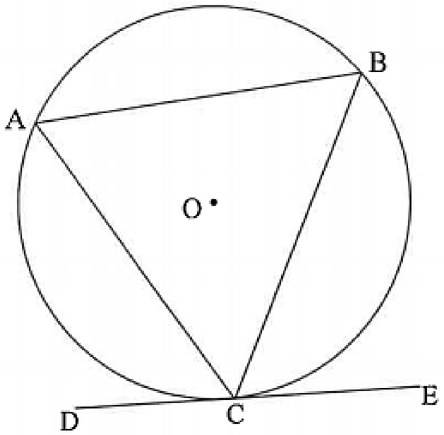
[8]

Euclidean Geometry

(May/June 2024)

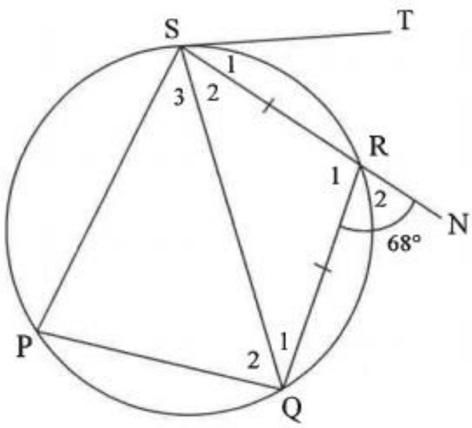
QUESTION 8

8.1 In the diagram, chords AB, BC and AC are drawn in the circle with centre O. DCE is a tangent to the circle at C.



Prove the theorem which states that the angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment, i.e. $B\hat{C}E = \hat{A}$. (5)

8.2 In the diagram, PQRS is a cyclic quadrilateral with RQ = RS. ST is a tangent to the circle at S. SR is produced to N. $\hat{R}_2 = 68°$.



Determine, with reasons, the size of:

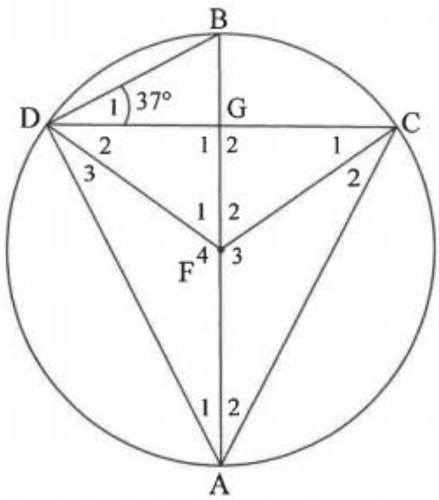
8.2.1 $\hat{P}$ (2)

8.2.2 $\hat{Q}_1$ (2)

8.2.3 $\hat{S}_1$ (2)
[11]

QUESTION 9

In the diagram, AB is a diameter of the circle, with centre F. AB and CD intersect at G. FD and FC are drawn. BA bisects $C\hat{A}D$ and $\hat{D}_1 = 37°$.



9.1 Determine, giving reasons, any three other angles equal to $\hat{D}_1$. (4)

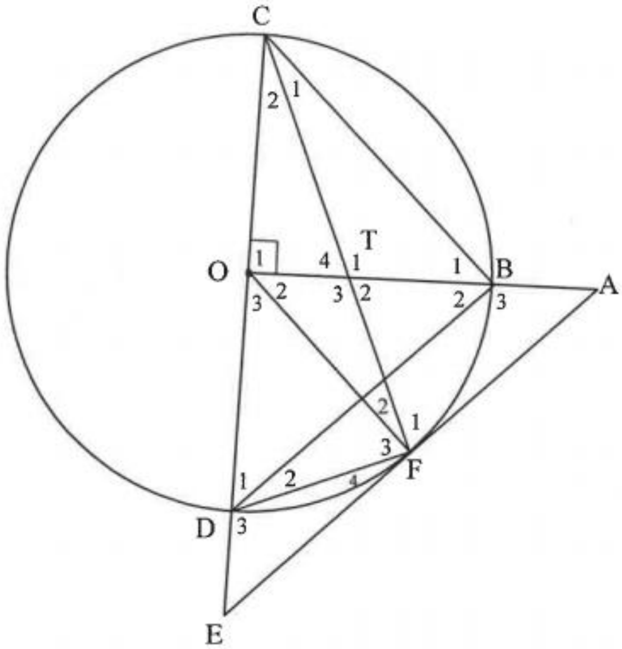
9.2 Show that DG = GC. (4)

9.3 If it is further given that the radius of the circle is 20 units, calculate the length of BG. (4)

[12]

QUESTION 10

In the diagram, COD is the diameter of the circle with centre O. EA is a tangent to the circle at F. AO $\perp$ CE. Diameter COD produced intersects the tangent to the circle at E. OB produced intersects the tangent to the circle at A. CF intersects OB in T. CB, BD, OF and FD are drawn.



Prove, with reasons, that:

10.1 TODF is a cyclic quadrilateral (4)

10.2 $\hat{D}_3 = \hat{T}_1$ (3)

10.3 $\Delta TFO \,|||\, \Delta DFE$ (5)

10.4 If $\hat{B}_2 = \hat{E}$, prove that DB $\parallel$ EA. (2)

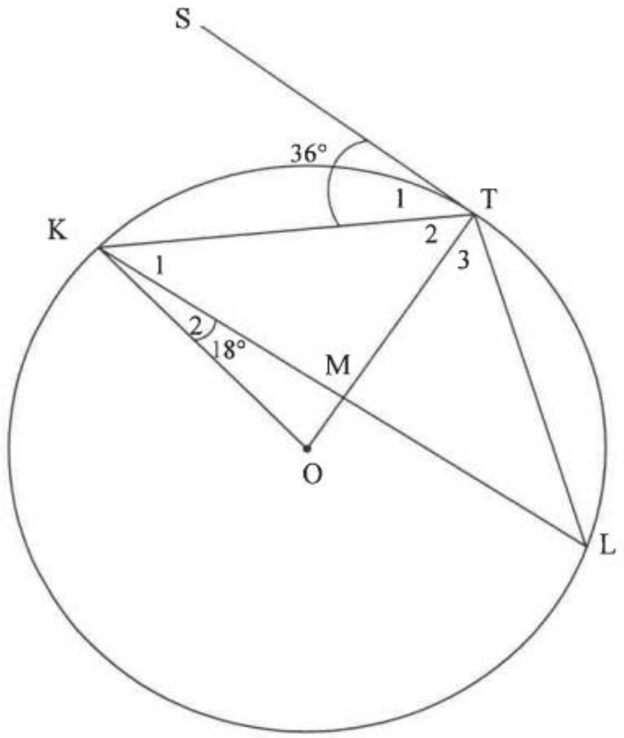
10.5 Prove that $DO = \dfrac{TO.FE}{AB}$ (5)

[19]

(May/June 2023)

QUESTION 8

8.1 In the diagram, O is the centre of the circle. K, T and L are points on the circle. KT, TL, KL, OK and OT are drawn. OT intersects KL at M. ST is a tangent to the circle at T. $S\hat{T}K = 36°$ and $O\hat{K}L = 18°$.



8.1.1 Determine, giving reasons, the size of:

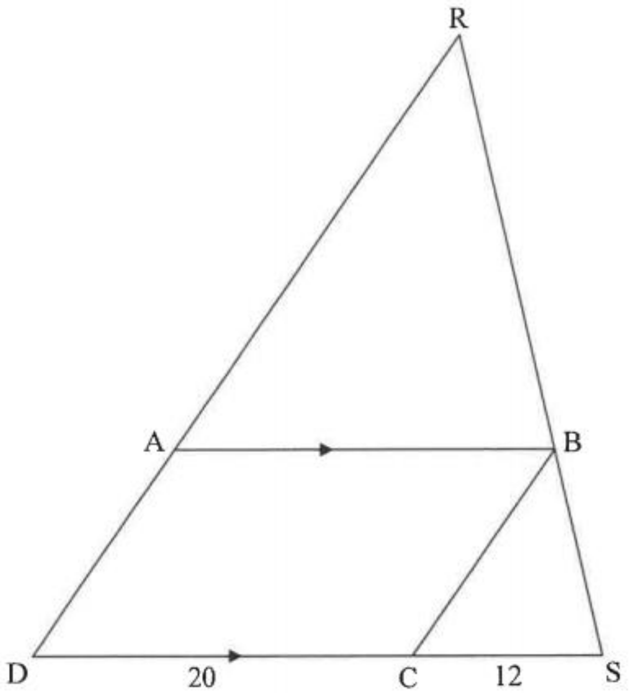
(a) $\hat{T}_2$ (2)

(b) $\hat{L}$ (2)

(c) $K\hat{O}T$ (2)

8.1.2 Prove, giving reasons, that KM = ML. (3)

8.2 In the diagram, ΔRDS is drawn. A, B and C are points on RD, RS and DS respectively such that $AB \parallel DS$ and $RB : BS = 5 : 3$. $DC = 20$ units and $CS = 12$ units.



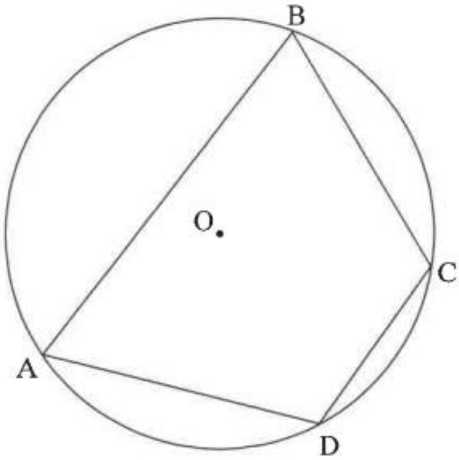
8.2.1 Prove, giving reasons, that $BC \parallel AD$. (3)

8.2.2 If it is further given that $RD = 48$ units, calculate, giving reasons, the value of the ratio $AD : AB$. (3)

[15]

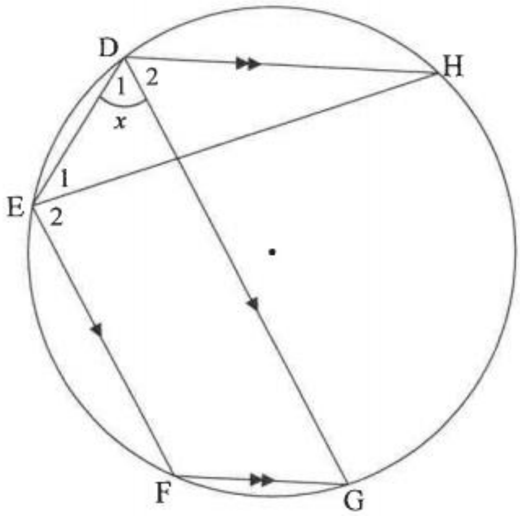
QUESTION 9

9.1 In the diagram, O is the centre of the circle. ABCD is a cyclic quadrilateral.



Use the diagram in the ANSWER BOOK to prove the theorem which states that the opposite angles of a cyclic quadrilateral are supplementary, that is prove that $\hat{B} + \hat{D} = 180°$. (5)

9.2 In the diagram, DEFG is a cyclic quadrilateral such that $EF \parallel DG$. H is another point on the circle such that $DH \parallel FG$. Chord EH is drawn. Let $\hat{D}_1 = x$.

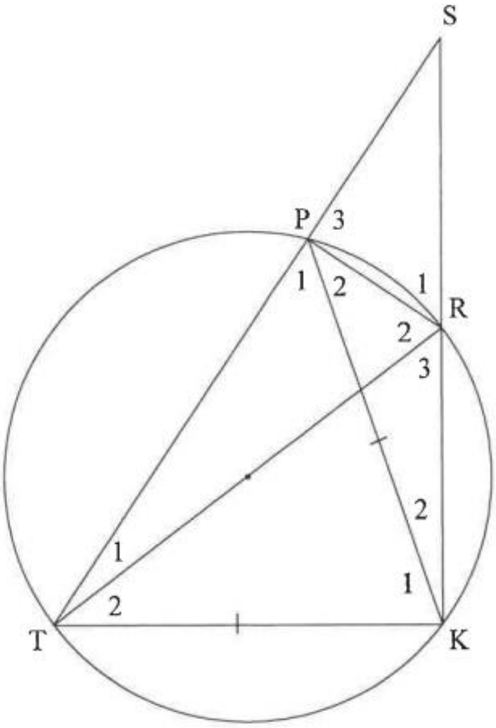


Prove, giving reasons, that $\hat{D}_1 = \hat{D}_2$. (4)

[9]

QUESTION 10

In the diagram, TR is a diameter of the circle. PRKT is a cyclic quadrilateral. Chords TP and KR are produced to intersect at S. Chord PK is drawn such that PK = TK.



10.1 Prove, giving reasons, that:

10.1.1 SR is a diameter of a circle passing through points S, P and R (4)

10.1.2 $\hat{S} = \hat{P}_2$ (5)

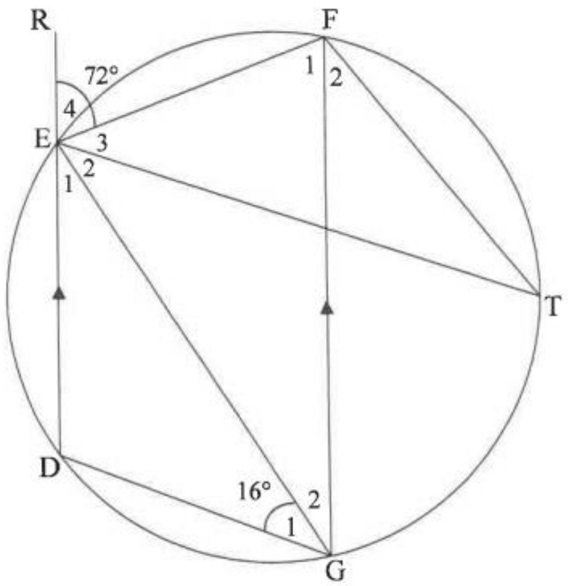
10.1.3 $\Delta SPK \parallel\!| \Delta PRK$ (3)

10.2 If it is further given that SR = RK, prove that $ST = \sqrt{6}RK$. (5)
[17]

(May/June 2022)

QUESTION 9

9.1 In the diagram, DEFG is a cyclic quadrilateral with $DE \parallel GF$. DE is produced to R. T is another point on the circle. EG, FT and ET are drawn. $\hat{E}_4 = 72°$ and $\hat{G}_1 = 16°$.



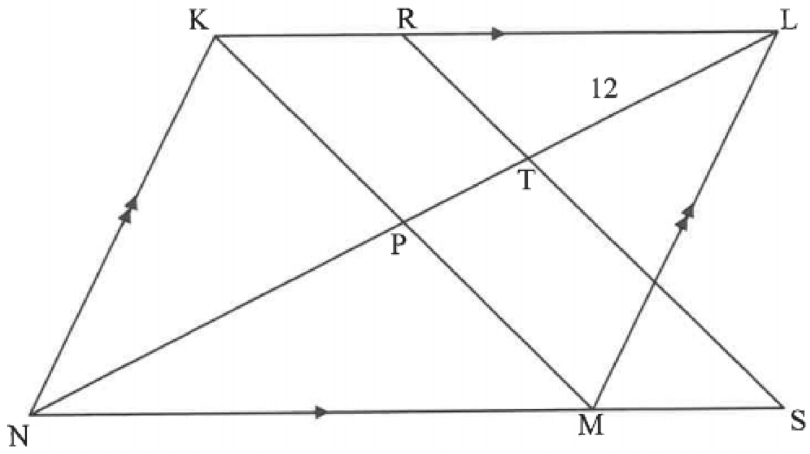
Determine, with reasons, the size of the following angles:

9.1.1 $D\hat{G}F$ (2)

9.1.2 $\hat{T}$ (2)

9.1.3 $G\hat{E}F$ (2)

9.2 In the diagram, the diagonals of parallelogram KLMN intersect at P. NM is produced to S. R is a point on KL and RS cuts PL at T.
NM : MS = 4 : 1, NL = 32 units and TL = 12 units.



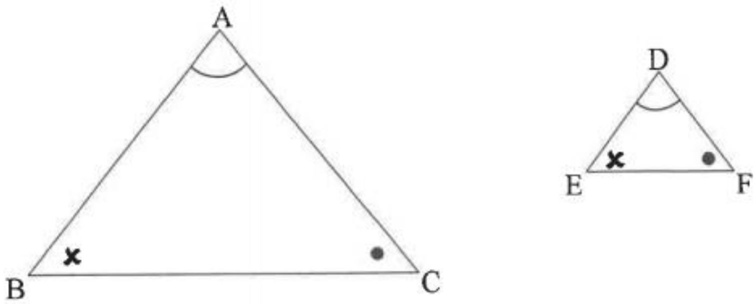
9.2.1 Determine, with reasons, the value of the ratio NP : PT in simplest form. (4)

9.2.2 Prove, with reasons, that KM ∥ RS. (2)

9.2.3 If NM = 21 units, determine, with reasons, the length of RL. (4)
[16]

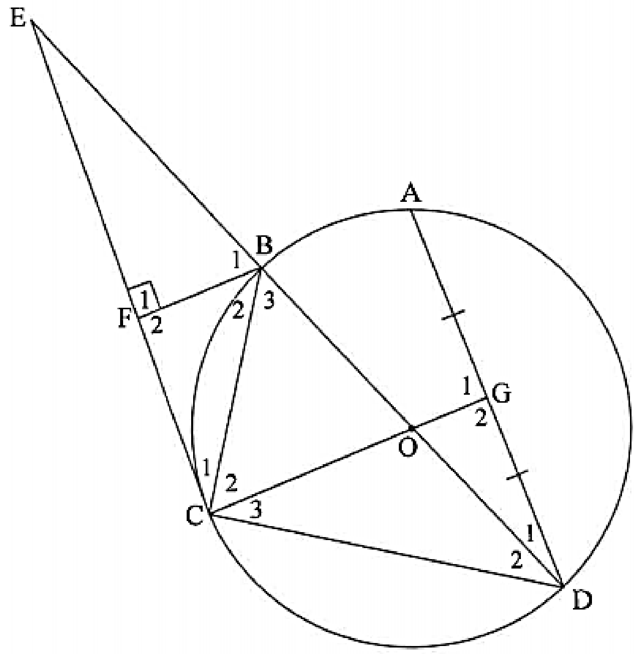
QUESTION 10

10.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e. $\frac{AB}{DE} = \frac{AC}{DF}$. (6)

10.2 In the diagram, O is the centre of a circle passing through A, B, C and D. EC is a tangent to the circle at C. Diameter DB produced meets tangent EC at E. F is a point on EC such that BF ⊥ EC. Radius CO produced bisects AD at G. BC and CD are drawn.



10.2.1 Prove, with reasons, that:

(a) $FB \parallel CG$ (3)

(b) $\Delta FCB \,|||\, \Delta CDB$ (5)

10.2.2 Give a reason why $\hat{G}_1 = 90°$. (1)

10.2.3 Prove, with reasons, that $CD^2 = CG.DB$. (5)

10.2.4 Hence, prove that $DB = CG + FB$. (5)
[25]

SECTION 5: INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

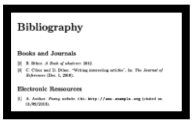
$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$



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